THE INFLUENCE OF DIFFERENTIALS IN CHILD MORTALITY BY AGE OF THE MOTHER, BIRTH ORDER, AND BIRTH SPACING ON INDIRECT ESTIMATION METHODS

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ABSTRACT

The objective of this investigation is to analyse the impact of differential mortality by birth order and age of the mother on the indirect estimates of child mortality. This indirect method was proposed by professor W.Brass and is based on reports about the number of children ever born and children surviving to women classified by age groups. The first step was to relax the constraints imposed on the method by the assumption that the risk of dying is invariant with birth order, mother's age and birth spacing patterns. To that effect, on the basis of the available evidence, a functional description of mortality by age of the child, which takes into account these differentials, was proposed. Then a beta-binomial probability distribution was used for describing fertility patterns by marriage duration and birth order, and a negative binomial distribution was adopted for describing nuptiality patterns. The models were tested using data from different countries and the results were satisfactory. All the necessary calculations to simulate proportions of children surviving (or dead) by age of the mother and number of children ever born were then executed on the basis of these three demographic models.

Birth distributions by age of the mother and birth order were obtained by compounding the fertility model by marriage duration with the nuptiality model. Then, under certain assumptions, mean

time-exposures to the risk of dying were calculated for children by birth order, current age of the mother, and parity. These exposures were combined with the functional description of mortality mentioned above, to yield proportions of children surviving by age and parity of the mothers. Adjusting factors by mother's age groups were calculated by relating these results to those obtained when mortality is assumed to be a function of the child's age only. These factors make estimates of mortality levels, obtained from reports from the younger mothers, comparable to the overall mortality for all children. They were applied to data from Peru and the results appeared to be very reasonable.

An important conclusion from the analysis of the average exposures to risk for children by mother's age and parity is that the exposures are fairly constant by family size, while the variation in the proportions of children surviving is significant. The practical implication of these findings is that variations in the proportions of children surviving are basically caused by differential mortality. The application of the technique was illustrated with two practical examples. Proportions of children surviving by family size and age of the mother from Bolivia, 1976 Census, and from Guatemala, 1970 Census, were analysed. An enormous differential in mortality by family size was observed in both countries. The patterns of the relative risks by family size were very similar in both countries.

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CHAPTER 1

The Development of Indirect Techniques for Obtaining Demographic Estimates.

I. THE DEVELOPMENT OF INDIRECT TECHNIQUES FOR OBTAINING DEMOGRAPHIC ESTIMATES.

1.1 Introduction

During the late fifties and the sixties the perception of the rapid population growth in most areas outside the developed world stimulated a growing interest in the study of the dynamics of the population, and how it affects and is affected by the economic and social structures. An increasing number of scientists and scholars from different disciplines directed their efforts toward a better understanding of the demographic phenomena. However, the situation concerning data sources required that more basic problems had to be tackled first. A direct measurement of demographic variables is obtained by relating the number of occurrences of vital events during a certain period of time to the population exposed to the risk in the same period. The population at risk is usually provided by censuses carried out at regular time intervals and the occurrences of vital events are recorded through vital registration systems. By 1950 few countries in the developing world had regular population censuses and less had complete and reliable registration systems. During the last three decades a remarkable improvement in the quantity as well as the quality of censuses has been observed. Many deficiencies still remain, omissions and distortions often hamper the calculation of conventional demographic indices, although in most cases tools for adjusting or correcting the data are now available. The problems concerning registration systems are less tractable. Progress here has been much slower and much remain to be done yet. The implementation of a registration system is a complex high-cost, long-term affair. In some developing countries the registration systems have reasonable completeness but cover only the urban or relatively more developed areas.

Confronted with this situation demographers have had to modify existing techniques for the estimation of demographic indices in societies where statistical information is incomplete or unreliable, develop new techniques to apply to data available in non-traditional forms, or develop new techniques to collect data quickly and cheaply and to obtain reliable demographic estimates by unconventional methods. Remarkable achievements have been obtained. However, the present situation is still far from ideal and considerable attention and efforts are required yet.

Some attempts to adapt procedures for obtaining direct demographic estimates are: i. introduction of additional questions in the population censuses in order to record the occurrence of vital events during a given reference period, stocks being provided by the same census; ii. execution of multi-round surveys that record number of vital events and time exposure to the risk in an area under observation through repeated enumerations; iii. dual record systems, where events are recorded by two systems, trying to maintain independence of both sources, and iv. retrospective surveys recording event histories like births histories and associated child deaths, marriage histories, and so on.

K. Hill (1975) criticised the first three approaches mentioned above mainly from the point of their use for estimating adult mortality, but most of his criticisms actually concern more general problems affecting such approaches, and they still apply to their use for other purposes such as estimating child mortality or fertility. We will discuss briefly some of these problems and then concentrate on unconventional approaches used to obtain indirect demographic estimates.

Under the assumption of independence in the probabilities of omission of the two sources, dual record systems provide a way for correcting the omisions after matching the events recorded by both systems. At the present dual record systems have lost the popularity that they enjoyed during the sixties. The procedure is too expensive and complex and independence between the two systems was proved to be very difficult to maintain.

The use of multi-round surveys for estimating fertility and mortality has also come under question since quicker results of good quality can be achieved from simpler and cheaper single round retrospective surveys. Nevertheless such an approach seems to be more useful for intensive studies, using small samples, related to a more specialised type of enquiry.

Extra census questions have some limitations arising from the problems that dating of events and age reporting present in statistically under developed societies. Some techniques have been devised to overcome such limitations, notably the P/F ratio method (Brass et al., 1968) and the Gompertz relational ratio method (Brass,1981, Zaba,1981) for estimating fertility, and a number of methods designed to deal with omission of reported deaths (Brass,1975, Brass,1979, Preston,1978, Preston and Hill,1979, Coale and Preston,1980) for estimating adult mortality. These techniques can be used for correcting information obtained from census questions as well as from registration systems. In favourable circumstances they have been successfully applied to information from either of these data sources.

The recording of event histories can provide rich data for the study of fertility and infant and child mortality. This type of demographic inquiry is very demanding in terms of organization and training of the interviewers. Lengthy and rather complex questionaires have to be carefully designed and executed. Those characteristics make this an expensive type of operation and impose some restrictions in the size of the samples to be used. For the purpose of estimating fertility and child mortality levels, trends and differentials, other types of enquiry, based on larger samples and few simple questions, can be used with advantage from a cost-efficiency point of view. The strength of event-history type of enquiries lies in the possibility of using individuals rather than aggregates as the units of analysis and the advantages that come from the grouping of events in their natural succession. These characteristics open very rich avenues for demographic research by allowing the use of more complex and promising theoretical frameworks and more sophisticated methodologies of analysis.

Another approach to get round the constraints imposed on the study of the population dynamics by data limitations has been the development of indirect techniques for obtaining demographic estimates. Indirect approaches to the estimation of demographic indices are based upon the effect of past events on some particular features of the population, rather than the relation of numbers of events in a period of time to stocks. These procedures provide estimates for demographic parameters from information not directly related to their values. The base of the indirect techniques is the construction of simple demographic models that can be specified by a few observable parameters. Under certain assumptions these models should be able to describe adequately the prevailing patterns and relationships among the relevant demographic variables. If those parameters can be easily estimated from information obtained from a few simple questions included in censuses or surveys, and the assumptions are more or less met or the measures are robust to some deviations from those assumptions, the advantages of this approach would be obvious. Based upon the models, estimations of relevant demographic parameters could be derived from information obtained through cheap and simple procedures. The experience of more than a decade of using these techniques demostrates their value through the large number of applications with very successful results. A significant amount of the current demographic knowledge of developing countries comes from applications of these methods. Undoubtedly the most successful development on this line has been the technique devised by Brass (1964) to obtain conventional life table measures of mortality from the proportions of children who have died among the total children ever born to women in different groups of ages.

1.2 Indirect estimation of infant and child mortality.

The proportion of children surviving among the total children ever born to women in a given age group obviously contains information on the level of mortality affecting those children. This kind of information was collected and the proportions used as an indicator of mortality for many years. However, those proportions are determined not only by the level of mortality but also depend on the length of time that the children have been exposed to the risk of dying. The mean time of exposure to the risk is equal to the difference between the mothers' current age and the mothers' age at birth of their children. Hence, the proportion of children dead will depend on the current age of the mother, the fertility distribution and the age pattern and level of mortality. W.Brass (1964) was the first to explore these relations systematically. He discovered that the relation between the proportion of children dead and the probability of dying before attaining certain exact childhood ages, q(x), is primarily influenced by the age pattern of fertility. It also depends on the age pattern of mortality, but not on the level of mortality. The dependence on the age pattern of mortality can be minimized by choosing the appropriate indicators q(x) to relate to each age group of respondents, leaving only the age pattern of fertility as the main factor influencing the relation. This relation was expressed as:

$$k_i = D_i \;/\; q(x) \qquad (1.1)$$

$i = 1,2,3,4,\ 5,\ 6,\ 7,\ 8,\ 9,10$

$x = 1,2,3,5,10,15,20,25,30,35$

The successive values of the subscript <u>i</u> indicates the ten successive five years age groups from 15-19 to 60-64. For calculating the k_i values, the age pattern of fertility was represented by Brass's polynomial fertility model (Brass,1968), which has a fixed shape but variable age location. The model of mortality was generated by the logit system from the general standard (Brass, 1968), and the stable age distribution for the women assumes a growth rate of 2 per cent per annum. The procedure was based on the assumptions of constant fertility and mortality over time. Another important assumption was that the risk of dying of a child is a function only of the age of the child and not of other factors, such as mother's age or the child's birth order.

Multipliers (k_i) were calculated for a range of fertility distributions specified by the parity ratios P_1/P_2, were P_1 represents the mean number of children ever born to women in age group 15-19 and P_2 similar average for women aged 20-24. The mean age of the fertility distribution was also specified. The appropriate k value for a particular application is found by interpolating between two tabulated values.

1.3 Sources of errors and robustness of Brass' estimating procedure.

An interesting framework for analysing the sources of errors and the robustness of the method has been provided by W.B.Arthur and M.A.Stoto (1983). For the subsequent analysis it is useful to make the following classification:

Concepts	Actual population	Model population	Survey population
- Probability of dying between birth and exact age a:	$q(a)$	$q^*(a)$	$q_s(a)$
- Relative frequency distribution of children at age a, born to mothers aged y:	$c(a)$	$c^*(a)$	$c_s(a)$

The multiplying factors k, in relation 1.1, were obtained as

$$k = q^*(x) \Big/ \int c^*(a)\, q^*(a)\, da \tag{1.2}$$

where the age x and appropriate limits of the integral change according to the ages of the women. The value $q(x)$ in the actual population is estimated by $\hat{q}(x) = k\, D_y$, where D_y is the proportion of deceased children among those born to women aged y, measured through the survey results: $D_y = \int c_s(a)\, q_s(a)\, da$.

The $\hat{q}(x)$ estimate, written in terms of the survey and the model functions, is:

$$\hat{q}(x) = \frac{q^*(x)}{\int c^*(a)\, q^*(a)\, da} \int c_s(a)\, q_s(a)\, da \tag{1.3}$$

A fundamental virtue of Brass's estimating procedure clearly appears in this expression: if the information from the survey is accurate and representative of the whole population, and the model functions correctly describe the fertility and mortality in the actual population, then both integrals cancel out in equation 1.3 and the estimate is exact. As the estimate depends on the relative distribution of children, it is affected only by the age distribution of fertility and not by the level of fertility. Furthermore, if mortality in the actual population differs from the model population by a constant scale factor, @ $q^*(x) = q(x)$, then the scale factor cancels out in k and the estimate is still exact. Hence, the model mortality does not have to represent the true mortality but only the age pattern. Arthur and Stoto analysed the effects caused on the estimate $\hat{q}(x)$ by errors in D_y, c^* and q^*. Errors in D_y, c^*, and q^* were represented as a differential or "small perturbation" from the true functions. Thus the differential of $\hat{q}(x)$ ($\delta \hat{q}(x)$) with respect to the pertinent function can be used as an approximate measure of the error in $\hat{q}(x)$ due to errors in D_y, c^*, q^* respectively.

The relative error in the estimate due to errors in the sample results, D_y, is

$$\frac{\delta \hat{q}(x)}{\hat{q}(x)} = \frac{\delta D_y}{D_y} \qquad (1.4)$$

that is, the proportional error in the estimates equal the proportional error in the sampling results.

As for the model mortality function, the relative error caused in the mortatity estimate will be:

$$\frac{\delta \bar{q}(x)}{\bar{q}(x)} = \frac{\delta q(x)}{q(x)} - \frac{\int c(a)\ \delta q(a)\ da}{\int c(a)\ q(a)\ da} \qquad (1.5)$$

It can be demonstrated that for a model mortality function with a different shape than the actual mortality, there is an age A for which the error is zero. Such age is equal to the average age of the children (currently alive or deceased) ever born to women aged y. If the age x to which the estimates refers is different from A, the translation is made along the model mortality pattern and will result in an error. Therefore the error caused by departure from the actual age pattern of mortality is minimized by choosing appropriates values x, for each age group of the women, that are close to the A values. Preston and Palloni (1977) showed that the closest x values to the A ones for some age groups differ in certain cases from the particular x values specified by Brass (although the difference is small), and the best choice is not independent of the "true" mortality pattern. In any case the relative errors will be more important for the very young ages, where the rate of change in the mortality function is higher.

Violation of the assumption of constant mortality over time will cause errors in the estimates, the current level will be over-estimated when mortality has been decreasing. Procedures to circumvent this problem will be discussed later.

Relative errors in the estimates caused by the wrong choice of the fertility model are measured through the following expression:

$$- \frac{\int \delta c(a)\ q(a)\ da}{\int c(a)\ q(a)\ da} \tag{1.6}$$

this type of error is not self cancelling. In order to fit the model accurately, the choice of the model fertility distribution is based on certain fertility indices observed in the survey population (i.e. P_1/P_2, P_2/P_3). However, for very young women the rate of change in the function c(a) is high and the denominator of the above error-expression is small, so estimates based on women under the age of 20 are sensitive to this type of error. Violation of the assumption of constant fertility will produce errors when the fertility model is specified by ratios between parities of different cohorts. If fertility has been decreasing the parity ratios will define a pattern of later fertility rather than the actual one. That implies a shorter exposure to the risk of dying than the one to which the children have been exposed, thus the level of mortality will be over-estimated. Some methods developed to deal with the problems introduced by fertility trends will be discussed later.

1.4 Early developments and applications of Brass' procedure.

Other authors proposed different procedures to estimate the set of multipliers k_i, although the theoretical bases were the same as in Brass' original approach. Sullivan (1972) used regression techniques instead of the tabular solutions for the k_i values. The multipliers were obtained by fitting estimating equations to data generated by a set of observed fertility schedules and the Coale-Demeny (1966) life tables. Trussell (1975) also used regression techniques and the Coale-Demeny life tables, but the fertility schedules were taken from the model fertility schedules developed by Coale and Trussell (1974). These different computational procedures do not provide substantially different results from those given by the original method. The use of Coale-Trussell fertility schedules improve on the polynomial fertility model, particularly for ages below 20, but other problems affecting the information from very young women make it of little use anyway. At the same time the introduction of the Coale-Demeny life table models provides more flexibility, but these potential advantage can only materialize when the age pattern of mortality in childhood is known, which is seldom the case in those countries where these techniques are most necessary.

The development of Brass' technique revolutionised the study of mortality under circumstances of limitated or defective data. In any of the three variants described above the method was massively applied to data from censuses and surveys until around 1978, when new developments of this method started to appear in the demographic literature. In those earlier approaches attention was focused on the information provided by women from 20 to 34 years of age. Estimates for q(2), q(3) and q(5) were obtained, then smoothed and combined to yield a unique consistent estimate of child mortality, usually expressed by q(2). In the light of later developments which relaxed the constraints imposed by some assumptions, this appears as a rather inefficient use of the information. However, at the time the method was created, the possibility of obtaining robust estimates of childhood mortality by very simple and cheap procedures opened a very fruitful avenue for research, stimulating and making possible numerous studies of child mortality at low cost in statistically under-developed countries. Indeed, a significant part of the present knowledge of the levels of chilhood mortality in those countries is the result of the application of these early approaches. A good example of successful exercise using these techniques is the I.M.I.A.L. programme (Behm et al 1975-1977). It consisted of a massive operation that covered most countries in Latin America, including a number of countries with satisfactory vital registration systems. For most of these countries the main contribution was that reliable estimates of child mortality were obtained for the first time. For other countries, with good registration systems, the inclusion of the necessary questions in the census were also largely justified; the results of the indirect estimates appeared in general to be in good agreement with the direct estimates, except in rural and relatively less developed areas within the countries. In such areas the registration systems were affected to some degree by omissions, and the indirect estimates helped to quantify these deficiencies.

Nonetheless, in the case of these countries with relative good data, the most important contribution came from the study of differentials in child mortality by a number of socio-economic and environmental categories related to characteristics of the mother, the father, the household or the communitty, information that is routinely collected in the censuses but is not recorded by the registration systems.

The use of these procedures in Africa and other parts of the world was met with equal success. Since these earlier stages, when only estimates for q(2), q(3) and q(5) were considered in the analyses, parallel improvements in the design of the questions, trainnig of the personnel, organization of the field work and refinement of the techniques of estimation have made possible a more comprehensive and efficient use of the data.

1.5 Recent developments of Brass-type estimation procedures.

As the quality of the data improved, it became clear that reliable estimates could also be obtained from information from older women. With more accurate data the need to relax some of the restrictions imposed by the assumptions on constant fertility and mortality was felt, as conditions of stability did not represent reality any more in most populations. Some approaches for adapting the procedures to changing fertility will be discussed first and then we will concentrate on the studies that adapted the method for applications under conditions of changing mortality.

1.5.1 Child mortality estimates under conditions of changing fertility

It was pointed out that changes in fertility may affect the estimates as the fertility model is fitted by using parity ratios based in two different age cohorts. One of the solutions suggested was to use the "true cohort" indices, when information on the number of children ever born is available from two censuses separated by intervals of five or ten years (K. Hill, H. Zlotnik and J. Trussell 1983). Coale-Trussell (1974) model fertility schedules and Coale-Demeny (1966) model life tables were used to generate data to which estimation equations were fitted by regression techniques, based on parity ratios for the true cohort. The main weakness of this approach lies on the assumption of comparable reporting in both data sources.

A different approach was suggested by Preston and Palloni (1977). They proposed to devise the distribution over time of the births to each cohort of women by matching children to mothers on census household records and using a reverse surviving procedure. If the age reporting is reasonably accurate the procedure would allow us to estimate the distribution of births over time without using any fertility models, avoiding the errors resulting from the estimation of such distributions and the problems arising from fertility changes. Like the "own children" method for fertility estimation (Cho, 1973), to which this approach is closely related, the disadvantages come from the problems of completeness of enumeration, children not living in the same household as their mothers, and other problems affecting a proper link between the children and their mothers. If these problems can be overcome the advantages of using the age distribution of surviving children to characterize the fertility history of each cohort are clear. In particular it would be most useful when: i. fertility trends are present in the population under investigation, ii. the fertility patterns in the population deviates markedly from normal patterns, and iii. in the analysis of differentials in child mortality levels among social classes or other permeable subgroups of the population for which parity ratios from different age cohorts do not describe the fertility history of a given cohort even under conditions of constant fertility over time. Among other calculation procedures, the following equation was suggested:

$$q(x) = D_i \{ A_i + B_i X_s + G_i c(2)\} \tag{1.7}$$

where A_i, B_i, and G_i are coefficients of the equation for the respondents' age group i, x is an appropriate age related to that cohort of respondents, X_s is the mean age at last birthday of surviving children to women in cohort i, and c(2) is the proportion of surviving children aged 2 or less last birthday. The procedure was then developed further by Palloni (1980), presenting equations to compute the time location of the estimates for respondents aged 15-19 to 40-44: $T_i = a_i + b_i X_s$. Naturally this equation would be necessary only if mortality has been changing, otherwise a time reference would be irrelevant. Procedures to deal with changing mortality are considered in next section, we mention this here as it is the only one specific for the estimation procedure based on the surviving children's age distribution. The following time location techniques concern approaches that use parity ratios as fertility distribution indices.

1.5.2 Child Mortality Estimates under Conditions of Changing Mortality

It is clear that the time reference for the estimates derived from older age groups of respondents are substantially different than those obtained from the younger ones. The question of time location became important as mortality started to decrease in most regions. Feeney (1976) was the first one to propose a solution to this problem. He showed that all consistent linear trends in period mortality tend to identify a unique level of infant mortality at a certain point in time prior to the census. Thus, under conditions of linear mortality changes, information on survivorship of children ever born to women in different age groups can be equated to mortality rates prevailing at different moments in time, the time location of the estimates being invariant with the rate of mortality change. An estimation procedure was later proposed (G.Feeney, 1980) to find tabular solutions for infant mortality rates and dates to which such estimates refer, from the proportions of children dead by age groups of the mothers. The fertility schedules were obtained by using Brass' polynomial fertility model and the mortality patterns were generated from Brass' general standard by using a one-parameter logit life table system. The use of infant mortality rates as a summary-index for childhood mortality levels presents some problems because of the sensitivity of such parameter to deviations from the underlying pattern of mortality in the observed population. Other procedures which are less dependent on the age pattern of mortality have subsequently been proposed .

Sullivan and Udofia (1979) demonstrated analytically that, under certain conditions, mortality estimates obtained from Brass-type procedures are equal to period mortality rates at some point in time, t^*_i, which does not depends on the rate of mortality change but only on the patterns of fertility and mortality. The t* values are obviously related to Feeney's empirical results. In this study the mortality function was represented by a standard age pattern of mortality, $d_s(a)$, multiplied by a level factor expressed as a function of the time, k(t). Assuming a constant annual rate of change in mortality: $k(t)=k_o(1-rt)$, where r is the rate of change. Then: $q^t(a) = k_o\,(1-rt)\; d_s(a)$.

The pattern of fertility, although unknown, is highly correlated to observable fertility indices, namely P_1/P_2, P_2/P_3. For a given pattern of mortality $d_s(a)$, the model to estimate t^*_i was then expressed as a function of the age group of the respondents and the fertility indices: $t^*_i = f^s_i(P_1/P_2)$; the function f^s_i has to be specified for each age group i and for the particular pattern of mortality.

An approach that considered time-period changes in mortality had been used by Coale and Trussell in 1977 (A.Coale and J.Trussell, 1977), when they first proposed a procedure for dating the Brass-type retrospective estimates. Coale and Trussell chained together the period levels in the Coale-Demeny life table models in order to derive cohort mortality for the children born to each age group of women. Brass (1983) has also developed a procedure for estimating the time location t^*_i, in this case the fertility distributions are derived from the Relational Gompertz Model. In Coale-Trussell and in Brass's time location procedures, the mortality, measured through a set of indicators q(x), still have to be expressed in terms of a unique parameter in order to make them comparable over time, so that mortality trends can be analysed. Dependence on the age pattern of mortality cannot be avoided but can be reduced by using a parameter other than infant mortality, for example q(5), as the level indicator for the whole series. The age pattern of mortality adopted for relating q(5) to mortality rates at other ages still will affect the results, but its effects would not be so strong as when infant mortality is used as the prime indicator of mortality level.

A different definition for the mortality function was adopted by Palloni (1979 and 1981). He analysed the effects of changing mortality by assuming cohort-mortality changes rather than time-period mortality variations. In this approach the Brass-type mortality estimate for each cohort and the time location define together the mortality level that affected each birth cohort of children born in such dates. Similar to Sullivan and Udofia, Palloni also represents the mortality function as the product of two components: $q(a,y) = f_y \, q_s(a)$; where f_y represents the changes of mortality in time, or from cohort to cohort, y indicating the date of birth for each cohort in terms of number of years previous to the census date; $q_s(a)$ represents the changes along the mortality function, according to a certain standard s, due to the effect of the child's age only. For y=a, $f_a q_s(a)$ gives the proportion of children who have died among those reaching exact age a at the census date. Hence, $f_a q_s(a)$ would represent a "multicohort" mortality function that would give the proportion of children dead born to a woman aged x when it is combined with the age distribution of the children, $c_x(a)$, born to that woman:

$$D_x = \int_0^{x-\alpha} c_x(a)\, f_a\, q_s(a)\, da \qquad (1.8)$$

α represents the earliest age at childbearing.

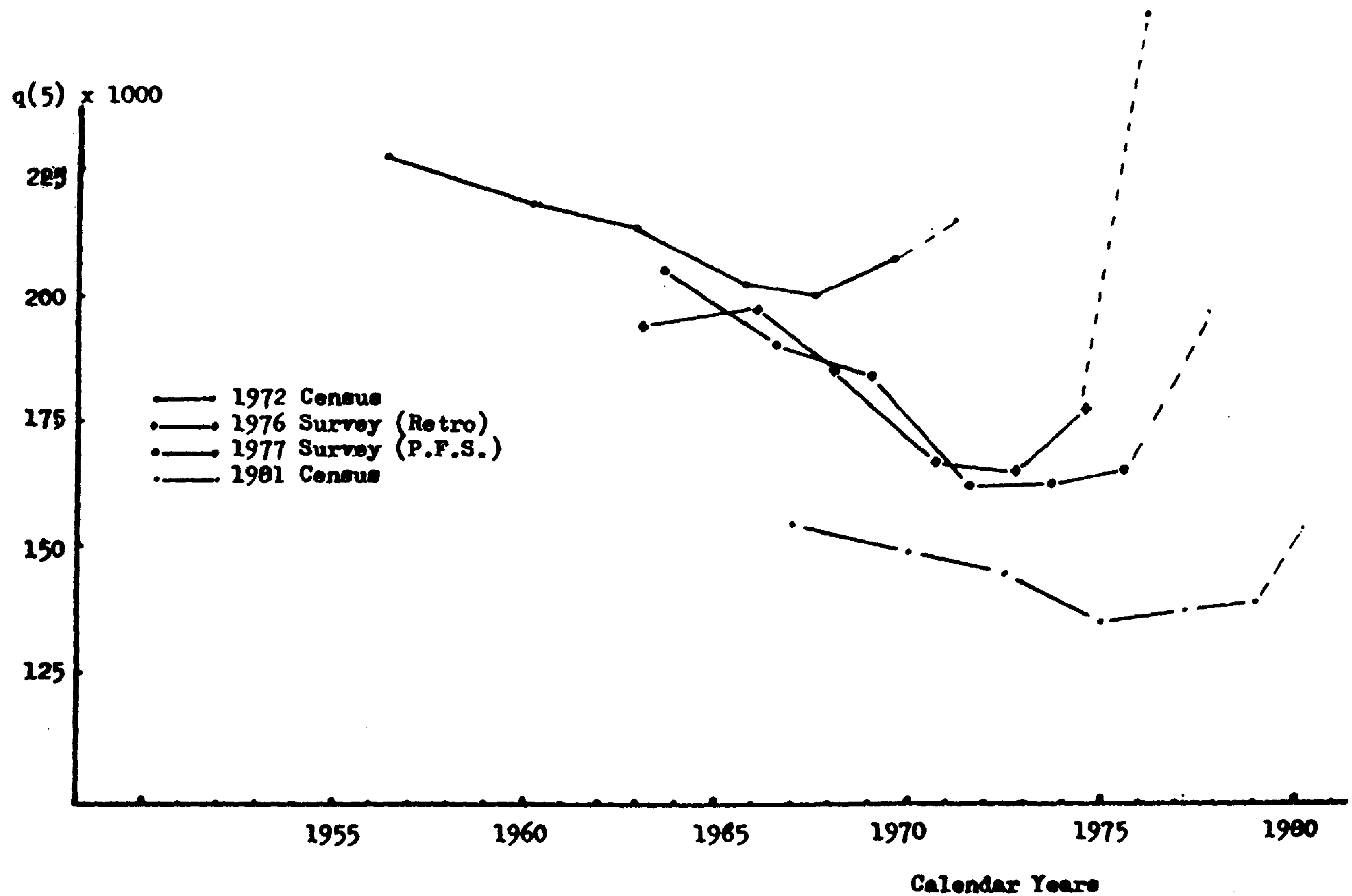
Palloni then assumes: (a) a linear change in mortality and (b) a quadratic change in mortality, estimating under those assumptions the time locations $t^*_i(a)$ and $t^*_i(b)$, which are interpreted as the number of years prior to the census or the age of the birth cohort for which the "multicohort" mortality function intercepts the "consistent" cohort mortality function, under the conditions imposed by the fertility distribution and the (a) linear, or (b) quadratic trends in mortality.

The application of the indirect techiques under conditions of changing mortality has increased the potentialities of the method enormously. These new developments have made it possible to study trends and differentials in mortality trends as well as levels. The main problem in these types of study does not lies in the reliability of the estimates, but in the relevance of the classifications adopted for analysing differentials, as Brass (1984) has pointed out. The robustness of the estimation technique has been confirmed by comparisons with results from other sources when they were available as well as by theoretical analysis (W. Arthur and M.A.Stoto,1983). The problem with the classifications arises from the fact that they are based on current characteristics of the women which might have changed since the death of the children and may not be relevant to the circumstances of those deaths. However, many of the characteristics of the women are already established by the time they enter adult life and change little during the period of their reproductive life. Hence, the problem of relevance of the classifications is not as accute here as it is in the case of indirect techniques for estimating adult mortality, for example, from information given by relatives.

A significant advantage has been that the procedures for the time location of the estimates do not require any additional questions. Since the method was first presented in the early sixties many censuses and surveys have collected the necessary information. Data from two, sometimes more, successive censuses are now available in many countries. In these circumstances the retrospective series of child mortality estimates can overlap in time, providing a very powerful tool for evaluation and analysis. It is clear that a second survey providing estimates comparable in time reference as well as methodology gives much more information than the simple addition of the two sets of data. The possibility of cross-checking the results expands considerably the strength of two overlapping retrospective time series. This is illustrated in figure 2.1. Four data sources provide the necessary information for Peru at time intervals that make possible the overlapping of the retrospective estimates. Consistent levels and trends of child mortality then emerge from different sources covering a period of about 20 years, leading also to the conclusion that the 1980 census data are affected by an omission of children who have died. Appropriate classifications also facilitate the analysis of differentials in levels as well as trends from these data (see for example Moser, 1983).

Another aspect that stands out in figure 2.1 is that estimates from the age group 15-19, and sometimes also 20-24, indicate higher mortality than the overall trend. This is related to the assumption that mortality is invariant with the age of the mother and the birth order. There is strong evidence that relative high fertility at very young ages of the women produce a combination of short intervals between births and young maternal ages that impair dramatically the children's chances of survival. Ewbank (1982) has considered the effect of birth order among other factors when he analyzed the sources of error in Brass's method, and produced improved estimates of child mortality in the case of Bangladesh. In that case he made corrections by ad-hoc procedures which were based on additional evidence from other sources. Apparently no attempt has been made yet to incorporate in the methodological basis of Brass-type estimation procedures the effects of mother's age and number of children attained on the risks of mortality in childhood. The possibility of dealing explicitly with such effects will be explored in this investigation. In the next section the main ideas will be outlined and the different aspects will then be developed in following chapters.

Figure 1.1: Trends in Childhood Mortality, Peru, 1955-1980



Source: Moser (1983), Table 14

1.6 Proportions of surviving children considering differential mortality by mother's age and birth order.

If mortality has been constant throughout the whole period during which the births occurred, and if there is no differential mortality by mother's age at birth, birth order, and total number of children attained by the women, then the proportion of children deceased among all children born to women aged i at a given census can be expressed, as it was seen before, as:

$$Q_i = \sum_{t>0} (1-L_t) \cdot c_i(t) \qquad (1.9)$$

where $c_i(t)$ is the proportion of children born during the t-th year prior to the census among all children born to women aged i at the census, and L_t is the proportion (of those children) surviving from birth up to the census date.

The information on the number of children ever born and the number of surviving children to women can be classified by age of the mother and total children ever born. Each age-parity group is characterized by a combination of a mother's age, a number of births of different orders and an implied average birth interval. With information broken down in this way it is possible to consider differential mortality by age of the mother at birth, birth order, and concentration of births, the latter being related to the number of children attained by women up to age i, thus indirectly taking into account the length of intervals between births. The proportion of children deceased for women aged i with total parity n is:

$$Q(i,n) = \sum_{j=1}^{n} \sum_{t>0} \{1 - L_t(j/i,n)\} \; c(t;\; j/i,n) \qquad (1.10)$$

where $L_t(j/i,n)$ is the proportion of surviving children of order j born to women aged i who have had n children in total; $c(t;\; j/i,n)$ is the distribution of those births over time. Expression 1.10 presents some complications, first it requires the specification of the mortality function taking into account all those differentials, then a fertility function by age and birth order is also required. These topics are developed in the following chapters, as indicated in the next section.

1.7 Contents of the following chapters

Chapter 2 deals with the problem of specifying the mortality function. The available evidence concerning the effects of mother's age, birth order and birth spacing on mortality during the early years of life is first analysed. On the basis of this evidence a functional description of mortality that takes into account those differentials is proposed. Chapter 3 deals with the fertility distribution by marriage duration and birth order. The viability of a discrete representation for the fertility distribution is tested using survey data from different countries. A nuptiality model is described in Chapter 4, and tested by fitting the model to data from several countries. Then the nuptiality model is compounded with the fertility model described in

Chapter 3, providing a distribution of births by order and age of the mother. The calculation process for obtaining proportions of children dead by age and parity of the mothers is the subject of Chapter 5. The process involve different stages; first the mean ages of the mothers at birth have to be obtained, then the mean time-exposures to the risk of dying are estimated, and finally, the mean exposures are combined with probabilities of survival to derive proportions of children alive. In Chapter 6 the "model" proportions of children dead (obtained under the assuption of differential mortality by birth order and age of the mother) are examined. The "model" proportions of children dead to women by age groups are then compared with the "expected" proportions (obtained assuming that the mortality for children ever born to women in any age group is the same, equal to the overall mortality for all children together), and the differentials by mother's current age are assessed. Adjusting factors to correct the retrospective estimates obtained from the younger age groups of respondents, in order to make them comparable to the mortality rates for all children, are obtained. Their application to real data is illustrated with an example, using data from Peru. Childhood mortality levels by mother's current age and parity are analysed in Chapter 7. First the average exposures to the risk of dying for children by family size (number of children ever born), within each age group of the mother, are examined. Then the possibility of studying differential mortality by family size from the retrospective information is discussed, and the analysis of empirical data is illustrated with two applications using data from Bolivia and Guatemala.

CHAPTER 2

Variation of Mortality with Age of the Mother , Parity and Birth Spacing.

II. VARIATION OF MORTALITY WITH AGE OF THE MOTHER, PARITY AND BIRTH SPACING

2.1 Introduction

In this chapter the studies that have dealt with the effects of age of the mother, her parity, and inter-birth intervals on the children's mortality risk, based on reliable data and big samples from statistically developed countries, are discussed first. The patterns of variation emerging from these studies are then compared with those observed in many other countries. This second group comprises those results from studies that, because of the smaller number of cases on which they are based, or for other reasons, appear less reliable than those from the first group. On the basis of this evidence an analytical representation for the effects of age of the mother, birth order and birth spacing (or birth concentration) is devised, in order to incorporate those differentials into a model life table. In other chapters this life table will be used for analysing the effects of such differentials on the Brass-type mortality estimates. The same mortality model will be used for developing a procedure to obtain indirect estimates of child mortality taking into account the total parity and the age of the mother.

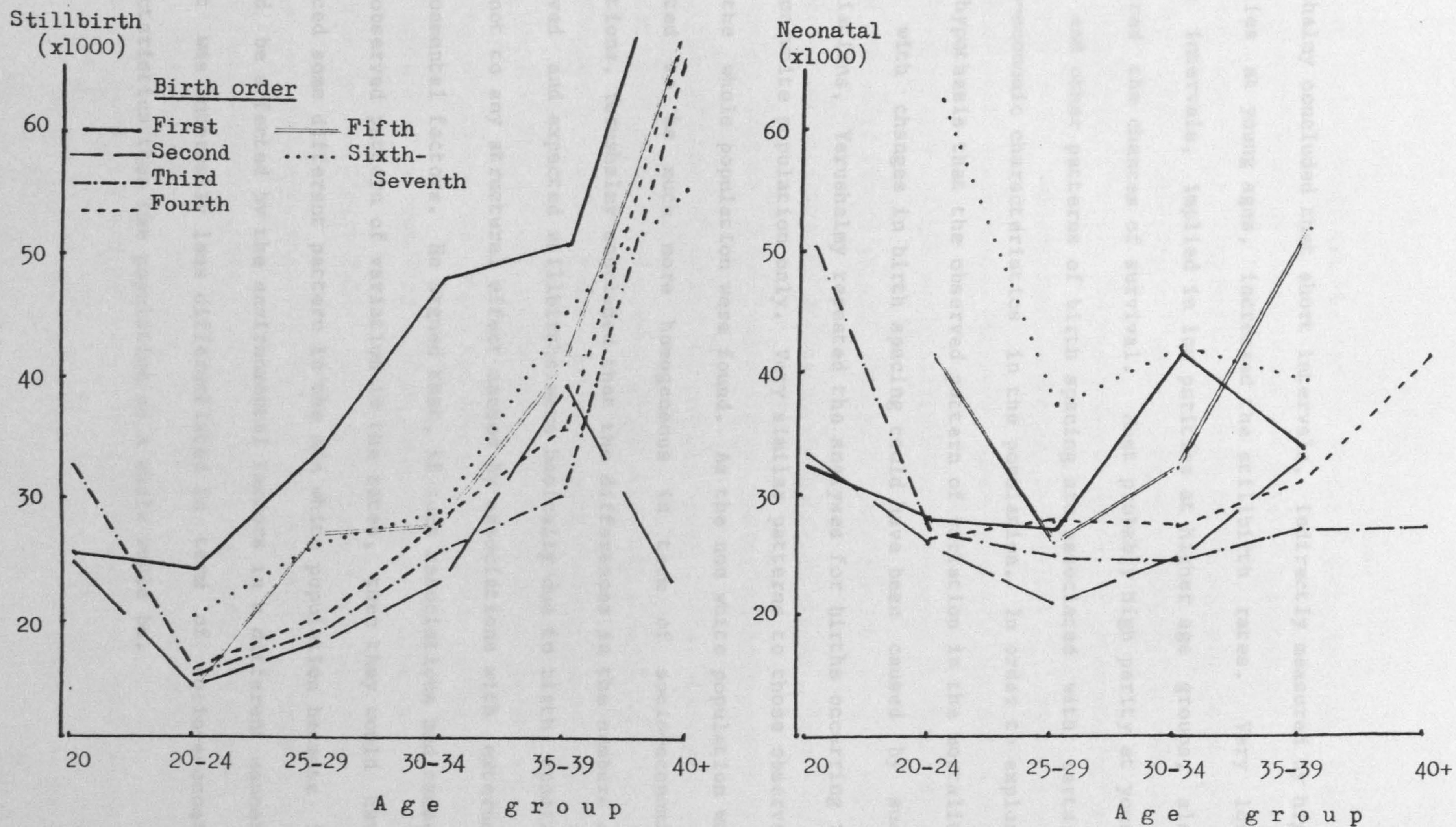
Before proceeding further it is convenient to distinguish between the term "parity order", which pertains to a woman and indicates the number of children she has born, and "birth order" which refers to a particular child and denotes the order the child occupies among all those born to the mother. Both terms are interchangeable when the mother's and the child's characteristics are observed at birth, and such is the case throughout the analysis carried out in this chapter. Obviously, when another birth occurs the mother moves to a higher parity, while the order of the previous child remains the same. In next chapters we will refer to women who, at a given age, have attained a certain number of children (parity order) and will be necessary to differentiate her children one from another by their birth orders.

2.2 The independent effects of age of the mother, parity and birth spacing on stillbirth, neonatal, and post-neonatal mortality

Although many studies had dealt with this subject before, the first statistically meaningful analyses of the patterns of variation of the stillbirth rate and neonatal death rate with parity and age of the mother, based on a big enough sample, were carried out by Yerushalmy (Yerushalmy, 1938, Yerushalmy et al,1940, Yerushalmy, 1945). Several studies had been published before, but were based on small samples. Yerushalmy (1938) first analysed the neonatal deaths and stillbirths that occurred in the New York State exclusive of the New York City in 1936. He found that neonatal death rates were high for first births, low for second and third births and then the rates gradually increased for higher births orders. As for age of the mothers, very young ages presented very high neonatal death rates, rates then decreased sharply to a minimum at about 27-28 years of age, and after that rose gradually with age of mother. Since there is a close association between age of the mother and birth order (first births occurred among the youngest mothers), he explored the possibility that the correlation between high rates for younger mother's ages as well as for first births were caused by such association. The author concluded that both factors had independent effects on neonatal mortality rates, such effects being apparent in the variation of the rates with one variable even after controlling for changes in the other variable.

The analysis of stillbirth rates showed broadly similar patterns, but the disadvantage of first births were stronger while high orders did not show as much disadvantage as in the case of neonatal mortality. With the exception of first births, birth order had little effect on stillbirths. Most of the variation appeared to be due to the age of the mother, where youth presented itself as a favorable factor for a live birth, as can be observed in figure 2.1. Some evidence of a birth spacing effect was also found, yet this factor was fully investigated by the author only later, using the births that occurred in the United States in the five year period 1937-1941 (Yerushalmy, 1945). Yerushalmy (1945) measured indirectly the effect of birth spacing by using a method of standardization known as "Westergaard's Method of Expected Deaths". The number of "expected" deaths, obtained under the assumption that the variation in the death rates is caused by the two factors (age and parity) operating independently, is compared to the observed number of deaths. The effect of birth spacing is then measured through the ratio of "expected" to observed deaths.

Figure 2.1: Stillbirth and neonatal mortality rates for legitimate births by mother's age and parity. New York State, 1936

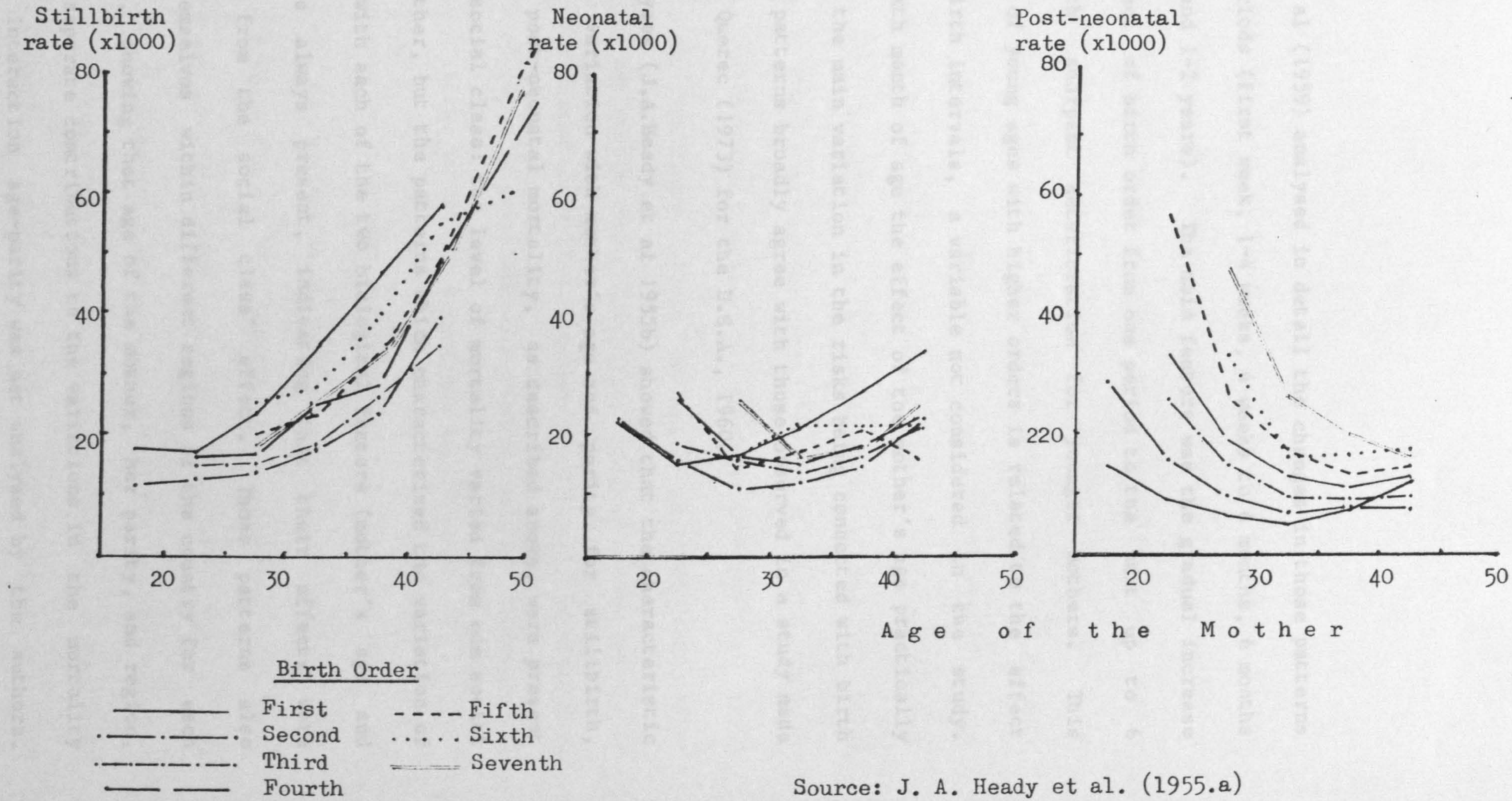


Source: Yerushalmy (1938), tables 7 and 8.

Yerushalmy concluded that short intervals, indirectly measured by high parities at young ages, increased the stillbirth rates. Very long birth intervals, implied in low parities at higher age groups, also impaired the chances of survival. Most probably high parity at young ages and other patterns of birth spacing are associated with certain socio-economic characteristics in the population. In order to explore the hypothesis that the observed pattern of variation in the mortality rates with changes in birth spacing could have been caused by such associations, Yerushalmy repeated the analyses for births occurring in the non white population only. Very similar patterns to those observed for the whole population were found. As the non white population was expected to be much more homogeneous in term of socio-economic conditions, Yerushalmy concluded that the differences in the numbers of observed and expected stillbirths were basically due to birth spacing and not to any structural effect caused by associations with external environmental factors. He argued that, if such associations had caused the observed pattern of variation in the rates, then they would have produced some different pattern in the non white population because it should be affected by the environmental factors in a different manner, as it was internally less differentiated in term of socio-economic characteristics than the population as a whole would be.

Another major study was carried out by the Social Medicine Research Unit (Medical Research Council) and the General Register Office, based on one and a half million children born in England and Wales during 1949 and 1950. The aims and methodology were described by J.N. Morris and J.A.Heady (1955). The variation of mortality rates with mother's age and parity was analysed separately for stillbirth, neonatal and post-neonatal death rates for about seven hundred thousand single, legitimate live births and stillbirths that occurred in England and Wales in 1949 (Heady et al, 1955a). Their results showed similar patterns for stillbirths and neonatal deaths as those described by Yerushalmy (figure 2.2): for any given parity stillbirth rates rose with age of the mother and for any age group rates increased with parity, except for first births, which presented a marked disadvantage in relation to second births; neonatal death rates increased regularly with parity for all age groups with the exception of first children born to mothers over 25 (which had higher rates than the second ones). The pattern of variation of post-neonatal rates differed from the two previous rates (figure 2.2): first orders presented the lowest rates except at ages more than 40; for a given mother's age post-neonatal rates increased with order and the younger the mothers the steeper the rise in those rates. The most distinctive pattern presented by post-neonatal rates is the steady decrease with age for all parities up to age 35. After age 35 the rates for lower orders increase a litle, particularly for first orders.

Figure 2.2: Stillbirth and infant-mortality rates for single legitimate births by mother's age and parity. England and Wales, 1949



Source: J. A. Heady et al. (1955.a)

Morrison et al (1959) analysed in detail the changes in those patterns for short periods (first week, 1-4 weeks, 4 weeks to 6 months, 6 months to 1 year, and 1-2 years). The main feature was the gradual increase in the effect of birth order from one period to the next up to 6 months, with sharpest deterioration for younger mothers. This interaction of young ages with higher orders is related to the effect of short birth intervals, a variable not considered in the study. After the sixth month of age the effect of the mother's age practically disappeared, the main variation in the risks being connected with birth order. These patterns broadly agree with those observed in a study made by Vavra and Querec (1973) for the U.S.A., 1960.

Other analyses (J.A.Heady et al 1955b) showed that the characteristic patterns of variation with mother's age and parity for stillbirth, neonatal and post-neonatal mortality, as described above, were present within each social class: the level of mortality varied from one social class to another, but the patterns which characterized the variation of the rates with each of the two biological factors (mother's age and parity) were always present, indicating that their effects were independent from the social class' effect. Those patterns also repeated themselves within different regions of the country for each type of rate, showing that age of the mother, her parity, and region, made their separate contributions to the variations in the mortality rates. The interaction age-parity was not analysed by the authors. However, its effects on the rates were evident, and particularly strong for young ages-high orders, revealing the detrimental impact of short birth intervals.

The data from the cohort of infants born in England and Wales during 1949-1950 provided enough information to study the pattern of variation in mortality with the spacing of births on a sound statistical basis. Although there was no direct information on the length of birth intervals, Osborne (1972) devised an indirect measure, that is an index of "birth concentration", by combining together the information on the number of previous births and the age of the mother. Osborne was then able to estimate simultaneously the independent effects of birth spacing, age of the mother and her parity, on the stillbirth, neonatal and post-neonatal mortality rates. His analyses revealed that birth spacing had a significant effect on child survival even after controlling for age of the mother and parity. Higher concentration of births (shorter birth intervals) appeared to be correlated with much higher risks of neonatal death and also higher risks of post-neonatal death, although in the latter the effect was less strong.

Osborne reanalysed the data used by Yerushalmy (1945) as his method of product factor standardisation improved on Yerushalmy's methodology. His results agreed in general with Yerushalmy's conclusions: high birth concentration was associated with much higher risks of stillbirth; the risk was also strongly correlated to the age of the mother and her parity. In an attempt to examine possible associations between these three physio-biological factors with socio-economic characteristics and the way such associations might influence the high correlations previously described, Osborne analysed data on live births and stillbirths that occurred in Scotland in social class III (as classified by the Registrar General for Scotland) during the period 1960-1967, and compared the results with those obtained for the whole population of Scotland for 1960-1964. The independent effects of maternal age, birth order and birth concentration were apparent in both social class III as well as in the whole country. The results were not identical but the curves were almost parallel on a semilogarithmic scale, implying that the proportional changes were very similar for both sets of data. Thus the patterns of variation in stillbirth rates with age of the mother, parity, and birth concentration, in social class III were analogous to those observed for all social classes together. Such results were in accordance with Yerushalmy's (1945) conclusions and with the findings of Daly, Heady and Morris (1955), suggesting that social class acted independently of the three physio-biological factors in its effects on the rates, and endorsing the hypotheses that the effects of such factors are independent from external environmental, social or economic factors.

All the studies above mentioned revealed closer similarities between the patterns of variation in stillbirth rates and neonatal mortality rates than between any of these two and post-neonatal mortality rates. However, neonatal mortality rates showed patterns of variation that can be considered as intermediate between the other two types of rates. These findings are hardly surprising, since during the first four weeks of life most of the infant deaths are still connected to the intra-uterine environment. During this period a high proportion of children die from causes that are congenital in character, and the problem of premature births ranks very high for neonatal mortality as well as for stillbirths. Some of the differences in the pattern of variation between periods seem to be related to the causes of death prevailing in the relevant period of life. For example, increases in the parity of the mother affect particularly the chances of survival of the infant during the post-neonatal period but have very little effect on the risks of dying during the first month of life. This seems to hinge on the fact that post-neonatal mortality is dominated by infectious diseases and the patterns of infant feeding, while such factors affect very little the risks in the neonatal period. The more children there are in the family the higher are the risks of catching infections as the opportunities for infection increase. At the same time, as the number of young children increases, they may start to compete with each other for the mother's attention and care, the family's resources, for food, and other needs.

Papavangelou (1971) analysed the independent effects of maternal age, birth order and birth concentration on the risks of infant deaths from seven groups of causes of death using data from England and Wales for the cohort born during 1949-1950. Causes such as immaturity, birth injury, congenital malformation, and asphysia and atelectasia showed a U-shaped effect with age of the mother while causes as respiratory diseases, enteritis and diarrhoea, and accidents presented a reverse J-shaped effect, with considerably higher risks at younger ages of mother. The effect of birth order was remarkably strong in the risk of infant death from enteritis and diarrhoea, the risk increasing sharply with birth order: birth orders higher than six showed risks more than five times those for second births. The risk of death from respiratory diseases also increased with birth order but after the sixth birth it remained at a level around three times higher than that observed for the second birth order. The risk of death from accidents did not show a regular pattern by birth order, but increased steadily with birth concentration. In general, Papavangelou's findings tend to support the hypothesis that the patterns of mortality variation with the three factors analysed here are linked to the structure of mortality by causes prevailing in the different periods of life. Hence the variation in such patterns along different periods of infancy seems to reflect the way these factors operate on the most severe causes of death within each period. The patterns of variation observed in the effects of maternal age, birth order, and concentration, for deaths by causes connected to biological factors resemble those patterns prevailing in the neonatal mortality rates, while for the group of causes linked to environmental factors they resemble the characteristics observed in the post-neonatal mortality rates.

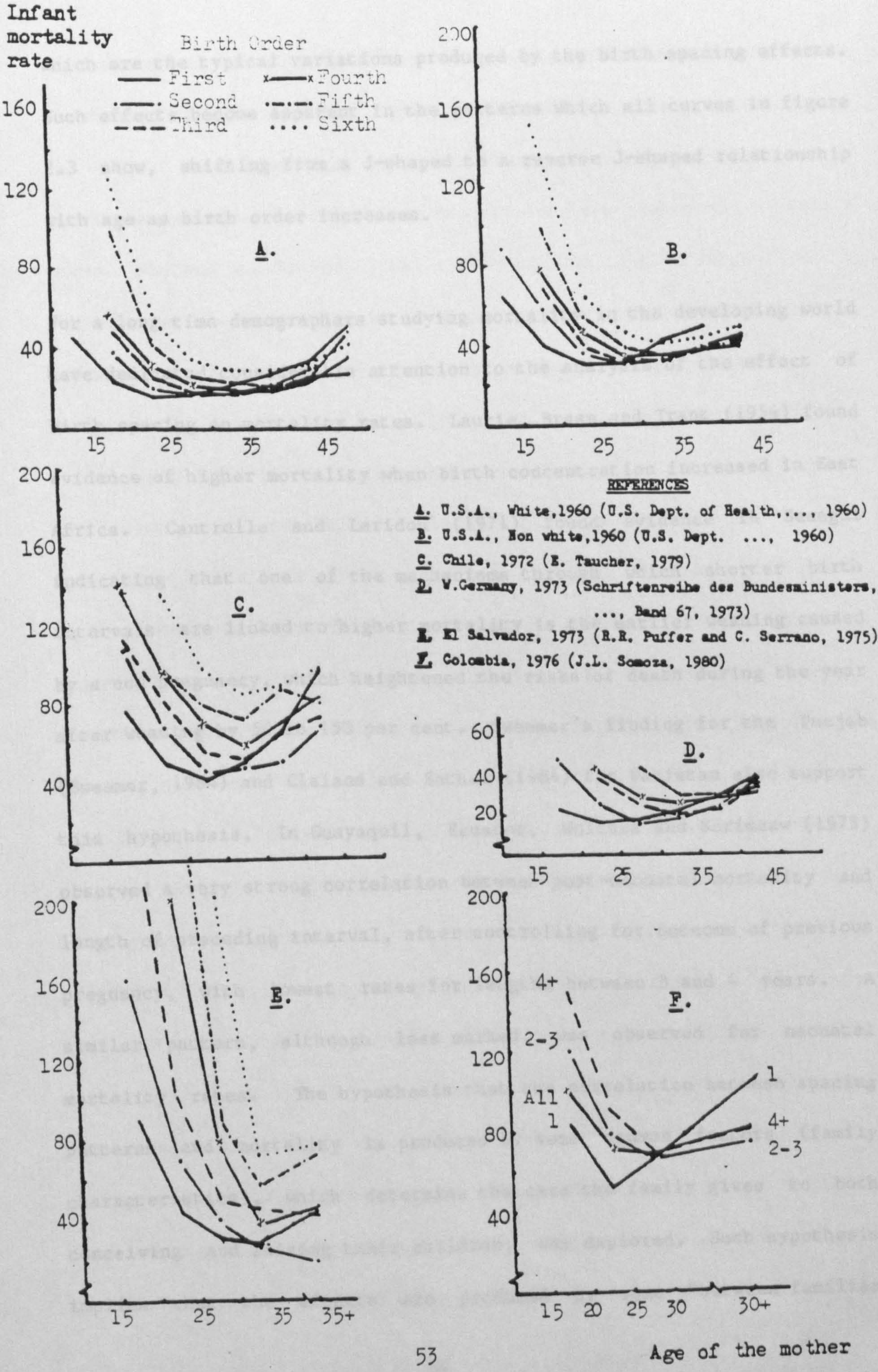
A very comprehensive review of the literature concerning the effects of parental age and birth order on pregnancy outcome and child development was done by Nortman (1974). Nortman used mainly secondary sources, considering those studies that would yield statistically meaningful results because of the experimental design and the sample size used in the investigations. On the assumption that relative risks by age remain much the same regardless of the absolute level of risks, age specific rates by birth order were converted into index numbers, based on an average (age group 20-34, generally) for each birth order equal to 100. For each risk the median age-specific index number was calculated and least square second degree polynomials fitted. The analysis of the patterns was then based on the smooth curves fitted to the median index numbers obtained from all the studies considered. There was a very wide differential in the absolute level of mortality among the different countries and among different regions or social classes within the contries examined. In spite of that, very narrow bands covered in most cases all the observed index numbers around the least square parabola, indicating the presence of a very similar basic pattern. The author concluded that the observations "support the hypotheses that biological processes are the chief determinants of the age pattern of reproductive risk and that social, cultural, and economic factors largely determine the degree of risk, whatever the mother's age".

The age-birth order patterns of mortality that emerged from Nortman's analyses agree with the results from those studies previously discussed. Indeed, such studies were included among the data analysed by Nortman. The J-shaped relationship between maternal age and still birth appeared very clear. For infant mortality, all births, the pattern with mother's age appeared as J-shaped; the break-down by age-birth order showed how the pattern shifted from a J-shape to a reverse J-shape as birth order increased. For higher birth orders the minimum risk emerged at older ages, bearing the typical effect of birth spacing, as Yerushalmy (1938) had pointed out.

2.3 Evidence on the patterns of mortality by mother's age, parity and birth spacing from developing countries' data.

The problems affecting vital registration systems in less developed countries have been discussed in the previous chapter. Because of those problems most of the available data for such countries have been obtained from surveys. Chile is an exception, there the registration system provides reliable information on birth order and other demographic and socio-economic characteristics for the infant and the infant's parents. A study of infant mortality was carried out by Taucher (1979), based on data for the cohort born during 1972. In spite of the enormous differential in the levels, the relationship between infant mortality rates and mother's age and parity described by Taucher resemble those patterns described in other studies for the United States, Germany, El Salvador and Colombia (figure 2.3). They are also similar to those discussed in the previous section, which is consistent with the hypotheses that the basic patterns are determined predominantly by biological factors, while the overall level depends on social, cultural and economic factors. In the case of Chile, Taucher's analyses also confirmed that the effect of the mother's age is stronger in the neonatal period, while the effect of her parity is particularly strong in the post-neonatal period. Birth spacing was not specifically analysed, although it was discernible that the increase in the rates with birth order was much stronger within the mothers' younger age groups and that the age of minimal risk increased with birth order,

Figure 2.3; Variation in Infant Mortality Rates with Age of the Mothers and Parity in Different Countries.



which are the typical variations produced by the birth spacing effects. Such effects become apparent in the patterns which all curves in figure 2.3 show, shifting from a J-shaped to a reverse J-shaped relationship with age as birth order increases.

For a long time demographers studying mortality in the developing world have dedicated considerable attention to the analysis of the effect of birth spacing on mortality rates. Laurie, Brass and Trant (1954) found evidence of higher mortality when birth concentration increased in East Africa. Cantrelle and Leridon (1971) found evidence in Senegal indicating that one of the mechanisms through which shorter birth intervals are linked to higher mortality is the earlier weaning caused by a new pregnancy, which heightened the risks of death during the year after weaning by 50 to 150 per cent. Sweemer's finding for the Punjab (Sweemer, 1984) and Cleland and Sathar (1984) for Pakistan also support this hypothesis. In Guayaquil, Ecuador, Wolfers and Scrimsaw (1975) observed a very strong correlation between post-neonatal mortality and length of preceding interval, after controlling for outcome of previous pregnancy, with lowest rates for lengths between 3 and 4 years. A similar pattern, although less marked, was observed for neonatal mortality rates. The hypothesis that the correlation between spacing patterns and mortality is produced by some common factors (family characteristics), which determine the care the family gives to both conceiving and raising their children, was explored. Such hypothesis implies that the effects are produced by some "between families differences", so they should not be present when those patterns are examined "within" families. Wolfers and Scrimsaw detected evidence of an effect within families (for which contrasting types of interval-survivorship were observed) in the case of post-neonatal mortality rates. Cleland and Sathar (1984) also concluded that "the relationship between length of preceding interval and survival of the index child is unlikely to be the spurious result of a common cause", according to their results for Pakistan. Analysing data from the Punjab, Sweemer (1984) did not reject the hypothesis of some influence of common factors affecting both child spacing and child survival. The studies for Guayaquil as well as for the Punjab revealed that short intervals not only affected survival of the following child, but also mortality rates for the previous child were heightened for the relevant period, when the child was followed by another birth after a short interval.

During the last few years many studies about the effects of maternal age, parity and birth spacing on the risks of mortality have been carried out. Most of them were based on data collected through the World Fertility Survey Programme, which provided abundant information for this type of study. Somoza (1980) presented an analysis of such data from the Colombian Fertility Survey. Although the sample size posed some restrictions, the pattern of variation with mother's age and parity order became apparent and was consistent with such patterns observed elsewhere (see figure 2.3). Thapa and Retherford (1982) observed in Nepal that infant mortality rates consistently increased with birth order when mother's age was controlled, whereas infant mortality decreased with age (controlling for birth order) up to age 35; older ages were not included because of small numbers and truncation. The effect of birth spacing was also analysed and the usual pattern of decreasing risks with longer intervals was encountered.

Rutstein (1983) analysed information on infant and child mortality from 29 countries covered by the W.F.S. programme. In an univariate analysis the U-shaped relationship between age of the mother and mortality rates was strongly evident for infant mortality but less strong for toddler (${}_1q_1$) and child mortality (${}_3q_2$). As for birth order, toddler and child mortality increased steadily with order; for infant mortality rates the pattern was less clear, but predominantly the risks increased monotonically with birth order, although in some cases first births presented higher mortality rates than second births. The analysis of inter-birth intervals revealed that "children born less than two years after the birth of their next oldest sibling are much more likely to die, even at ages over one year".

The effects of the birth spacing patterns on the risks of mortality were analysed from a multivariate approach by Hobcraft, McDonald and Rutstein (1983), using W.F.S. data from 26 countries. Infant mortality risks rose dramatically when a birth had been preceded by any previous birth in an interval of less than 2 years. The occurrence of births in both periods 0-2 years and 2-6 years previous to the index birth heightened considerably not only the risks of infant but also toddler mortality and it showed some deleterious effect in child survival as well. For toddler and child mortality rates the authors analysed the effects of births in the 0-17 and 0-30 months following the birth of the index child. Toddler mortality rose dramatically when the index child was succeeded by another birth within 18 months. Child mortality risks also increased almost universally with a birth following in less than 30 months. Patterns of short inter-birth periods either because of preceding or succeeding sibling were always detrimental for the survival of the index child in any of the periods of life analysed, and a succession of short intervals heightened the risks substantially. Control by mother's education was introduced, but not by age of the mother or birth order. Although mother's age and birth order would most probably account for some of the differences, the authors concluded that there is little doubt that the pattern of birth spacing affects the chances of survival of children born at both ends of the interval. In their analysis for Pakistan, Cleland and Sathar (1984) observed that the effect of birth spacing remained after controlling by age of the mother and her parity. However, their analysis raised doubts on the assertion that there is any cumulative effect of successive short intervals over the childbearing career of a woman, a hypothesis suggested by other results (Puffer and Serrano, 1975), and supported by the finding of Hobcraft et al. In the case of Pakistan the length of the immediate preceding birth interval appeared as the crucial factor. It should be pointed out, however, that Cleland and Sathar used a different approach in their analyses: length of the two immediate preceding intervals and average length of all previous intervals, while Hobcraft et al. considered counts of births in two-years-time segments.

Trussell and Hammerslough (1983) analysed W.F.S.'s data from Sri Lanka using hazard models. The same method of analysis was applied by Martin et al. (1983) to data from Philippines, Pakistan and Indonesia, arriving at similar conclusions with regard to mortality patterns by age of the mother, birth order and birth spacing. The main effect shown by the models was the typical U-shaped pattern of mortality with mother's age; control by socio-economic variables increased the negative impact of mother's young ages. Considering birth order, first births and births 2-3 had the lowest risks; the univariate analysis in some cases indicated higher mortality for first births than for second and third births, but when control by maternal age was introduced first births always appeared with the lowest mortality. When length of previous interval was combined with birth order, controlling for mother's age, it was clear that the risk for a given birth order increased as the length of the birth interval decreased. The highest risks were observed for higher orders preceded by short intervals.

2.4 Summary

The deleterious effect on child survival of young ages of the mother at birth appeared clearly in all studies. Ages above 35 have also a negative impact on child survival. The results are not so conclusive in relation to the independent effects of birth order. There is little doubt that as birth order increases from second, and particularly third orders, mortality risks increase. Some evidence suggests that these effects extend beyond the first year of life. Considering first births, there is an interaction with age of the mother; as age of the mother increases, mortality risk for first births increases more than the average risk for all other orders does. For young maternal ages (under 25), first births have lower relative infant mortality than higher orders, although the latter frequently appears influenced by the effect of short birth intervals. Most of the attempts to separate out the effects of birth interval from birth order and maternal age have excluded first births from the analyses because of the methodological problem posed by the lack of a previous interval. Besides, higher birth orders at young ages are necessarily related to short birth intervals. In some multivariate approaches a category of birth spacing that would comprise all cases of first births and exclude almost all other cases was defined (i.e. no births in the last six years, in Hobcraft et al.,1983), but the sample size made it difficult to introduce simultaneous controls by birth order and maternal age. When such controls were introduced, first orders appeared always with the lowest risks, although not very different from second and third orders, and clearly with better survival chances than higher orders.

Concerning birth intervals, although the adverse effect of very short intervals may appear in some cases overstated because of over-representation of premature births (with much higher mortality risks) in this category, there is no doubt that births preceded by short intervals face higher mortality risks during the first year of life. There are also indications that the harmful effect of short birth intervals extends beyond the first year of life. Some evidence, although not conclusive, suggest that a very long birth interval also impairs the child's chances of survival. Some studies provided evidence contrary to the hypothesis of a cumulative deleterious effect on survival for children born after a succession of short birth intervals. More research into this topic would be necessary before accepting these results as conclusive.

Another feature observed in these studies was that the effect of maternal age weakened as child's age increased, with very little impact after the first year of life. Birth spacing and parity order still affect the child's chances of survival after the first year (probably through the "competition" factor and through increased opportunities for infections).

2.5 A basic pattern of infant mortality by age of the mother, birth order, and birth spacing.

As was discussed in the previous chapter, indirect estimation of mortality taking into account parity order and age of the mother would require the specification of mortality risks by maternal age, parity, and birth spacing. The kind of data used for these estimation procedures do not allow a measure of birth interval as such. Birth spacing patterns have to be incorporated through some index of birth concentration, by combining age and birth order.

On the bases of the evidence analysed in previous sections, and the work done by Nortman and Osborne, the probabilities of survival (or death) by age of the child, mother's age, birth order, and birth concentration are obtained for ages under one year on the assumption that they can be approximately described by the product of a factor representing the overall level of mortality (K), a function representing the pattern of variation by age (x) of the child (which can be characterized by a standard life table, $l_{sd}(x)$), and three factors representing the patterns of variation by age of the mother (A(y)), birth order (P(r)), and birth concentration (C(c)), respectively:

$$ {}_{x}q_{0} = K \{ 1-[l_{sd}(x)]\} \; A(y) \; P(r) \; C(c) \qquad 0<x<1 \qquad (2.1) $$

Maternal age ceases to have any relevant effect for ages over one year.

The effect of parity order and birth concentration on ${}_1q_x$ is assumed to decrease linearly, disappearing after age four:

$$ {}_1q_x = K * \{1-[l_{sd}(x+1)/l_{sd}(x)]\} * \{1+(1-x/4)\ [P(r)-1]\} * $$
$$ * \{1+(1-x/4)\ [C(c)-1]\ \}; \qquad x=1,2,3 \qquad (2.2) $$

$$ {}_{x-4}q_4 = K\ \{\ 1-[l_{sd}(x)/l_{sd}(4)]\ \} \qquad x>4 \qquad (2.3) $$

The standard $l_{sd}(x)$ can be represented by any appropriate model, in this case Brass's general standard will be used. Categories of birth concentration were formed by combining five year age groups and parity order of the mother. An arbitrary category of birth concentration was defined for the first birth order. The values for A(y), P(r) and C(c) are given by third degree polynomials. These functions were obtained by fitting the polynomials to a set of multipliers which, when applied to the overall rates, allowed us to reproduce (closely enough for the purposes of this study) the different sets of infant mortality rates by mother's age and birth order, available from different studies. Starting from the patterns described by Osborne, the multipliers were subsequently adjusted to give an average pattern which approximately resemble the variations observed in several countries. The multipliers were then standardized so that, when the specific mortality rates are applied to a particular birth distribution, they would reproduce the overall mortality level represented in the standard. The distribution of births used for standardizing the multipliers was obtained from a model of fertility by age and birth order, which is described in

Chapter 4. It represents a situation where the mean age at first marriage is about 20 and the total fertility rate about 5, which seems to be the average case for countries where the procedures developed in this study might be applied. Given a different birth distribution, the relative frequencies of births in categories of higher or lower risk would produce an overall mortality somehow higher or lower than the standard. Such variations will not affect the validity of the results obtained in Chapter 6, as they are accounted for in the calculation procedure. (The calculation procedure is described in detail in Chapter 5).

Equations 2.4, 2.5, and 2.6 express the functional dependence of child mortality on maternal age (y), birth order (r), and birth concentration (c). Age is measured as age last birthday (completed years) rather than exact age. In equation 2.4 the age scale is measured in units of 5 years, with origin at 12. Thus, complete years of age 17, 22, 27, etc, are indicated by values of y equal to 1, 2, 3, etc.

$$A(y) = 1.96 - 0.8109\, y + 0.1725\, y^2 - 0.00944\, y^3 \qquad (2.4)$$

$$P(r) = 1.247 - 0.312\, r + 0.0817\, r^2 - 0.0045\, r^3 \qquad (2.5)$$

$$C(c) = 1.18 - 0.31636\, c + 0.07967\, c^2 - 0.003973\, c^3 \qquad (2.6)$$

Table 2.1 presents the categories of birth concentration corresponding to each combination of birth order and age of the mother at birth.

Table 2.1: Categories of birth concentration defined by combining birth order with mother's age at birth.

Age of mother	Birth order									
	1	2	3	4	5	6	7	8	9	10
15-19	3	6	8	9	10	10				
20-24	3	5	6	7	9	10	10	10		
25-29	2	3	4	5	6	7	8	9	10	10
30-34	1	2	3	4	5	5	6	7	8	9
35-39	1	2	3	3	4	4	4	5	5	5
40-44	1	2	2	2	3	3	3	3	3	3
45-49	1	2	2	2	3	3	3	3	3	3

An explanation of table 2.1 would be in order, since the interpretation of these categories is not quite stright forward. The categories in this table were obtained by adapting Osborne's ideas to the requirements of the present study. Working with vital statistic data, Osborne first defined an indirect measure of the inverse of birth interval for all births excluding first orders:

-If women experiencing their first births are excluded, the number of birth intervals a mother has experienced is one less than the order of the last recorded birth (or the women's parity order). Let B_{ij} be the number of births occurring in maternal age group i and birth order j. Then the mean number of birth intervals experienced by a mother in age i, m_i, is:

$$m_i = \{ \sum_j B_{ij} * (j-1) \} / \{ \sum_j B_{ij} \} \qquad (2.7)$$

-A measure of birth concentration (c_{ij}) was then derived by calculating the ratio of the number of intervals experienced by a mother in a particular age group, i, to the average number of intervals for mothers of that age:

$$c_{ij} = (j-1)/m_i = \{(j-1) * \sum_j B_{ij} \} / \{ \sum_j (j-1) * B_{ij} \} \qquad (2.8)$$

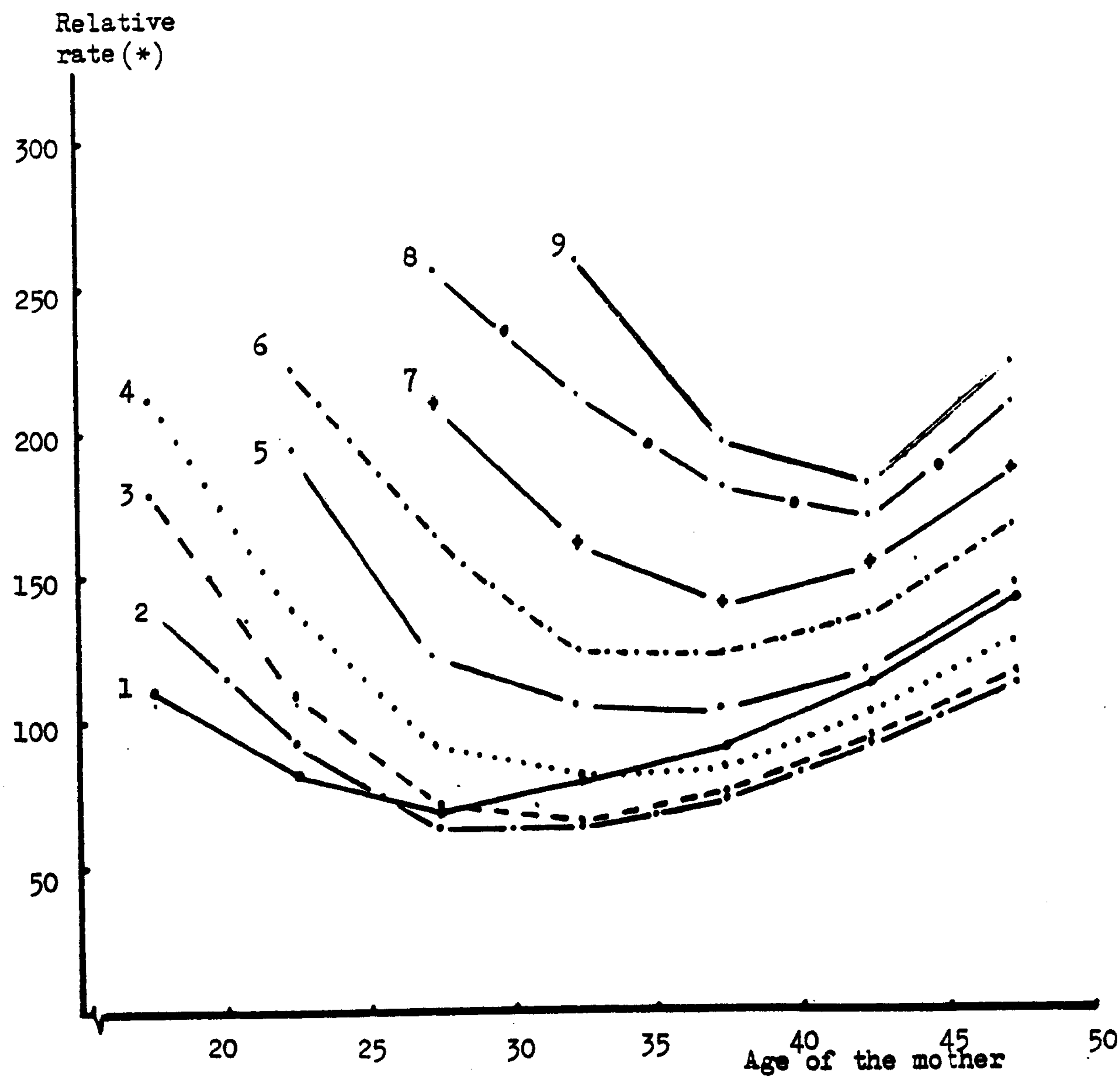
For a given birth order, birth concentration decreases as maternal age increases. Contours of constant birth concentration follow paths involving simultaneous increases in both maternal age and birth order. Osborne then broke the range of birth concentration values into several intervals, so each maternal age-birth order subclasses could be allocated to a birth concentration group. For the purposes of this study, an arbitrary category of birth concentration was allocated to first births. Aiming to representing (with reasonable approximation) the paths followed by observed mortality rates for first orders by age of the mother, the "effect" of this arbitrary category was assigned by trial and error.

The functional representation of mortality defined in this chapter is consistent with the hypothesis that there is a basic pattern of mortality in the early ages, determined by biological factors such as age of the child, age of the mother at birth, birth order, and length of birth interval, while the degree of risk (overall level of mortality) is determined by environmental, social, cultural, and economic factors.

The patterns of infant mortality by mother's age, birth order, and age-birth order, defined by this model, are illustrated in figure 2.4.

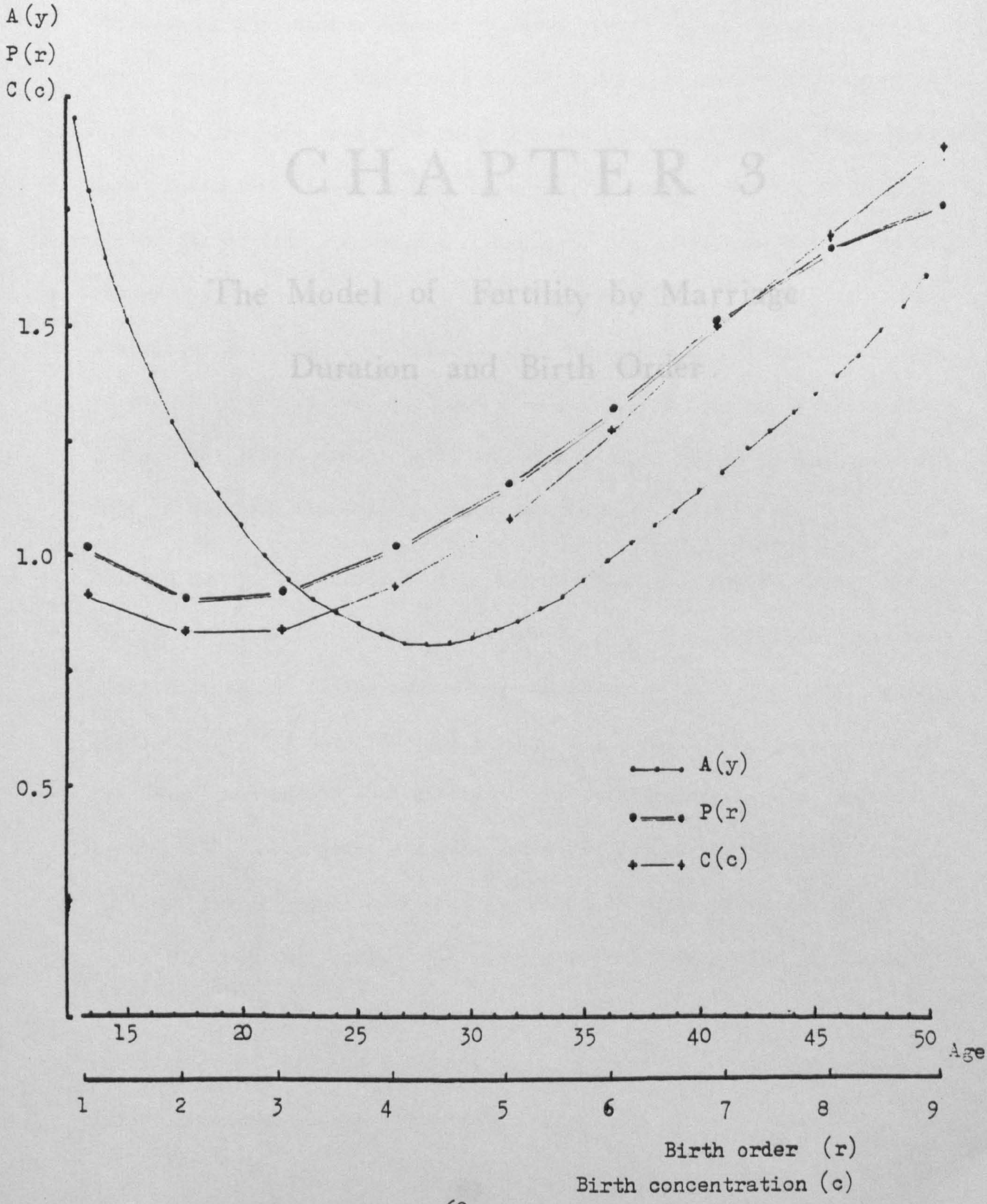
Figure 2.5 shows the patterns of variation in the factors A(y), P(r), and C(c), as determined by the polynomials.

Figure 2.4: Infant mortality pattern by mother's age and birth order, according to the model representation.



(*)"Relative" in the sense that, according to the model, the absolute level of mortality depends on the scale factor K. In this case the factors A(y), P(r) and C(c) were applied to a standard rate of 100 per thousand, with K=1.

Figure 2.5: Effects of age of the mother, A(y), birth order, P(r), and birth concentration, C(c) on the mortality function.



CHAPTER 3

The Model of Fertility by Marriage Duration and Birth Order.

III. THE MODEL OF FERTILITY BY MARRIAGE DURATION AND BIRTH ORDER

3.1 Introduction

Farahani (1981) quotes a paper by Powys (1905) as one of the earliest works on modelling distributions of births by order and marriage duration, in this case, by using Pearson type functions. Since then many scientists have worked on modelling human fertility from a biological or from a demographic approach. An exhaustive and in depth review of the work done in this field is not the concern of this investigation. However, because of the amount of research or the advances in theory that followed from them, the work by Henry (1953, 1957, 1961, 1972) and by Davis and Blake (1956) should be mentioned as some of the most significant contributions.

For the particular purposes of this investigation, we are loking for a simple mathematical model that would describe approximately the distribution of births by order and marriage duration in a given population. The main ideas of a very simple model which has sought to represent adequately such distributions were presented in a paper by Brass (1970). The model was then developed further by Farahani (1981). In the second section of this chapter the characteristics of this fertility model are discussed. The analytical formulation of the model is presented in the third section, and in following sections the model is fitted to observed distributions of births by order and marriage duration and the results discussed.

3.2 The basis of the fertility model

Only a brief description of the model, with some comments concerning its use for our particular purpose, are presented here. The characteristics of the model are discussed in more detail by Brass (1970) and Farahani (1981).

Even in a population where no family planning is practised, fecundability varies between women and for each woman it changes with age. In a drastic simplification we can assume that fecundability is constant among women and remains invariant over their whole reproductive life. The restriction on constant fecundability among women will be relaxed later. Constancy with age is not a serious limitation for the purpose of this study. It is clear that fecundability starts to decline before a woman become permanently sterile. However, declines in fecundability with age only become relevant over certain ages after which the relative impact of fertility on the kind of analysis performed here is very small. As for adolescent sub-fecundity it would be equivalent, in its effect, to a lower proportion of married women at such ages, and it can be dealt with through the nuptiality function, which will be analysed later. Under these circumstances the assumption that fecundability is constant over the whole reproductive life is not consequential on our results.

The time interval from one pregnancy to the next is determined mainly by the pregnancy duration, the non-susceptible post-partum period, and the level of fecundity. The first component is invariant; in natural fertility the second component is also largely invariant for a given society, as it is strongly determined by physiological factors and social control (mainly expresed through norms concerning breastfeeding and post- partum sexual abstinence). Once a woman enters the susceptible period, the next pregnancy will follow a period of delay with length depending on the fecundability (probablity of a conception in a menstrual period), the frequency of sexual intercourse and a chance component. As stated above, fecundability can safely be assumed constant for the range of ages that cover the most relevant period, for our purposes. For this period the frequency of sexual intercourse is not expected either to introduce much variation in the delay to next pregnancy. As for the chance factor, it is expected to produce a random variation with most cases concentrating around the average delay period, the length of the birth interval being then largely determined by predominantly invariant components.

Under those conditions it is possible to determine an appropriate length of time-interval such that it would be impossible that two births occur in the same period, and that the occurrence of a birth in an interval is independent on whether of not a birth has happened in the previous one. Foetal deaths and reduced non-susceptible post-partum periods due to neonatal deaths obviously disturb this picture, violating the assumptions on which the model is constructed, with regard to an individual woman. However Farahani's analysis showed, by comparing the model results to computer simulated distributions which included such sources of variations, that these variations effectively average out in the whole population. In the next section, where a model is fitted to real data, we can see that the results do not appear to be seriously distorted by the simplifying assumptions.

The assumptions on which the model is based can then be itemised as follows (Brass,1982.b):

1) Marriage duration can be divided into interval-units of an appropriate length such that within each interval only one birth can occur, and the probability of a birth in an interval is independent of when other births occur.

2) The proportion of women at risk, that is those who are able and willing to have r or more births, depends only on r, and is described by a stopping rule function, S(r).

3) Each woman at risk has a probability "p" of having a birth in an interval, whatever the marriage duration or birth order.

4) The probabilities "p" are distributed over the women according to a Beta distribution with parameters a and b.

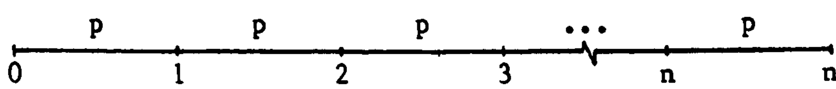
The model, as defined above, implies that the pace at which women at risk move to higher parity orders is given by the average of the probabilities p over all women, and is constant for all orders. It means that distributions of birth intervals from the previous birth (or from marriage, given an appropriate starting point), are the same whatever the birth order.

Considering whole populations rather than individual couples, the restrictions imposed by the assumptions enumerated in the previous paragraph do not seem to be as strong as they might appear at first sight. Indeed, in most societies the lengths of birth intervals are fairly constant with birth order, except for the last intervals which are a little longer. Such variation for higher orders is connected to the decline in fecundability (and perhaps coital frequency), and appears consequential in the family building proccess only at later ages or at very long marriage durations. Furthermore, some studies (i.e. in Hobcraft and McDonald, 1984) have revealed a surprising uniformity in the pattern of birth intervals in a substantial number of countries which are very different in most other respects.

As for the constraints of unchanging birth interval distributions, different authors have stressed the remarkable similarities of such distributions, found in different populations (Farahani,1981, Brass,1982.b, Pellizi,1982, Penhale,1984, Ford,1981), which implies that this feature is not far removed from reality.

3.3 The fertility model by marriage duration and birth order

Under the assumptions described in the previous section, natural fertility can be represented by independent births in time, with probability "p" that a birth will happen in a given time interval from marriage. It appears that this can be made approximately true by choosing the appropriate length of the time interval, so that the probability of a birth in an interval is independent on whether or not a birth has happened in the previous one. From Farahani's work the appropriate length seems to be between 18 and 24 months, however at this stage we do not need to adopt a fixed length and the point will be considered later. Thus, marriage duration can be divided in successive intervals each having the same probability of a birth "p":



n = number of completed intervals units from marriage.

Set $b_n(r)$ as the probability of a woman having r births in n intervals and $B_n(r)$ that of having r or more in n intervals. Then, $b_n(r) = B_n(r) - B_n(r+1)$. We can impose now a restriction due to family planning or sterility, and denote by S(r) the proportion of women who will be able and willing to have r births or more. This is independent of n (number of intervals from marriage) and will depend only on the number of births already attained. Thus, $\pi_n(r) = B_n(r)\, S(r)$ is the probability of r or more births in n intervals, and $D_n(r) = \pi_n(r) - \pi_n(r+1)$ is the probability of r births in n intervals under the conditions imposed by the "stopping rule" S(r).

Under conditions of equal probabilities of occurrence in each interval and independence of the events, the probability of r births in n intervals follows a simple binomial probability distribution:

$$b_n(r) = \binom{n}{r} p^r q^{n-r} \quad ; \text{ where } q=1-p;\ r=0,1,\ldots,n \qquad (3.1)$$

For a particular woman, "p" (probability of her having a birth in one interval) will depend on her fecundability. If we consider that in any population fecundability varies between women, in the model representation we can allow for this by considering that p varies over women according to a probability function, say "beta", in the whole population:

$$f_B(p) = [\ p^{a-1} \ (1-p)^{b-1}\ ]/\ B(a,b) \tag{3.2}$$

where $0<p<1$; a and b are positive constants; and

$$B(a,b) = \int_0^1 x^{a-1} \ (1-x)^{b-1} \ dx \tag{3.3}$$

The Beta distribution was chosen after analysing empirical data by using computer simulations (Brass, 1970). Then, allowing for heterogeneity in the population, the joint frequency distribution of r and p for a fixed number of intervals n is given by $b^*_n(r)$:

$$b^*_n(r) = \int_0^1 f_B(p) \binom{n}{r} p^r q^{n-r} \ dp \tag{3.4}$$

that is, the probability of a woman having r births in n intervals from marriage, with fertility parameter equal to p, multiplied by the probability that p will assume a certain value in the population where fecundability varies between women according to a density function $f_b(p)$. Hence, $b^*_n(r)$ represents the probability of r births in n intervals in a population where fecundability varies among women. Integration over p gives:

$$b^*_n(r) = \binom{n}{r} B(r+a,n-r+b) \ /\ B(a,b) \tag{3.5}$$

where B(a,b) represents a Beta function (in this case with parameters a and b) and so does B(r+a,n-r+b), with their respective parameters indicated within the brackets. Hence, in terms of the gamma functions, $b^*_n(r)$ can be written as:

$$b^*_n(r) = \binom{n}{r}\{\Gamma(a+r)\ \Gamma(b+n-r)\ \Gamma(a+b)\}\ /\ \{\Gamma(a+b+n)\ \Gamma(a)\ \Gamma(b)\} \qquad (3.6)$$

We need to estimate the number of births of r-th order occurring during the (n+1)th interval in a population under the stopping rule S(r). Women susceptible to having an r-th birth in interval (n+1) are those who have attained r-1 children in the n previous intervals: $b_n(r-1)$; and, according to the stopping rule, only a proportion S(r) of these women are exposed to such risk. Then, the proportion exposed multiplied by the probability of a birth, p, gives the probability of an r-th birth occurring in interval n+1:

$$D_{n+1}\{r/(r-1)\} = S(r)\ b_n(r-1)\ p \qquad (3.7)$$

or, allowing for variation in fecundability among women:

$$D^*_{n+1}\{r/(r-1)\} = S(r)\int_0^1 f_B(p)\ b_n(r-1)\ p\ dp \qquad (3.8)$$

integrating over p,

$$D^*_{n+1}\{r/(r-1)\} = S(r)\ \binom{n}{r-1}\ B(a+r,n+b-r+1)\ /\ B(a,b) \qquad (3.9)$$

Putting aside for the moment the stopping rule, calculations are very easy after simplifying the gamma functions in the following relations:

1) The ratio of the probabilities for the same birth order in two

successive intervals:

$$D^{*}_{n+1}\{r/(r-1)\} \; / \; D^{*}_{n}\{r/(r-1)\} = n\,(b+n-r) \; / \; [(n-r+1)\,(a+b+n)] \qquad (3.10)$$

That is, the probability of an r-th birth in the (n+1)th interval is equal to the probability of r births during the n preceding intervals multiplied by a factor which depends on the number of intervals and the parameters a and b.

2) $$D^{*}_{r+1}\,[(r+1)/r] \; / \; D^{*}_{r}\,[r/(r-1)] = (a+r) \; / \; (a+b+r) \qquad (3.11)$$

the probability of an (r+1)th birth in the (r+1)th interval is equal to the probability of r births during the preceding r intervals multiplied by (a+r)/(a+b+r), where a and b are known because they are the parameters of the distribution and r is the number of intervals.

3) The probability of a birth in one interval (average of the parameters "p" for each woman in the whole population) is a/(a+b).

Then, $$D^{*}_{1}(1/0) = a/(a+b) \qquad (3.12)$$

From equations 3.12 and 3.11 it is possible to calculate the upper diagonal of a worksheet which presents the distribution of births by duration of marriage and birth order. The rest of the table is obtained by using equation 3.10. Table 3.1 illustrates such calculations.

In order to test the flexibility of the model for describing different situations, it has been fitted to W.F.S. data from different countries. The main purpose of this exercise is not to obtain the best fitting of such data, but to evaluate how reliable this model can be for representing a wide range of variations in the pace of family formation. So far the model have been used for studing some European data (Brass,1982, Pellizi,1982, Penhale,1984). Although the model should be most useful for evaluating and analysing data from countries with high fertility, lack of suitable data have prevented a wider use of the model, and it has not been tested in such situations yet.

Table 3.1: Probabilities of a woman having a r-th birth in the n-th interval from marriage (Parameters a=3.5,b=3.5)

Interval	Birth Order					
n	1	2	3	4	5	6
0-1	0.500					
1-2	0.2188	0.2813				
2-3	0.1094	0.2188	0.1719			
3-4	0.0602	0.1477	0.1805	0.1117		
4-5	0.0355	0.0985	0.1477	0.1422	0.0762	
5-6	0.0222	0.0667	0.1128	0.1333	0.1111	0.0540
6-7	0.0145	0.0462	0.0846	0.1128	0.1154	0.0872
7-8	0.0099	0.0327	0.0635	0.0916	0.1058	0.0981
8-9	0.0069	0.0237	0.0479	0.0733	0.0917	0.0960
9-10	0.0050	0.0175	0.0366	0.0584	0.0774	0.0877

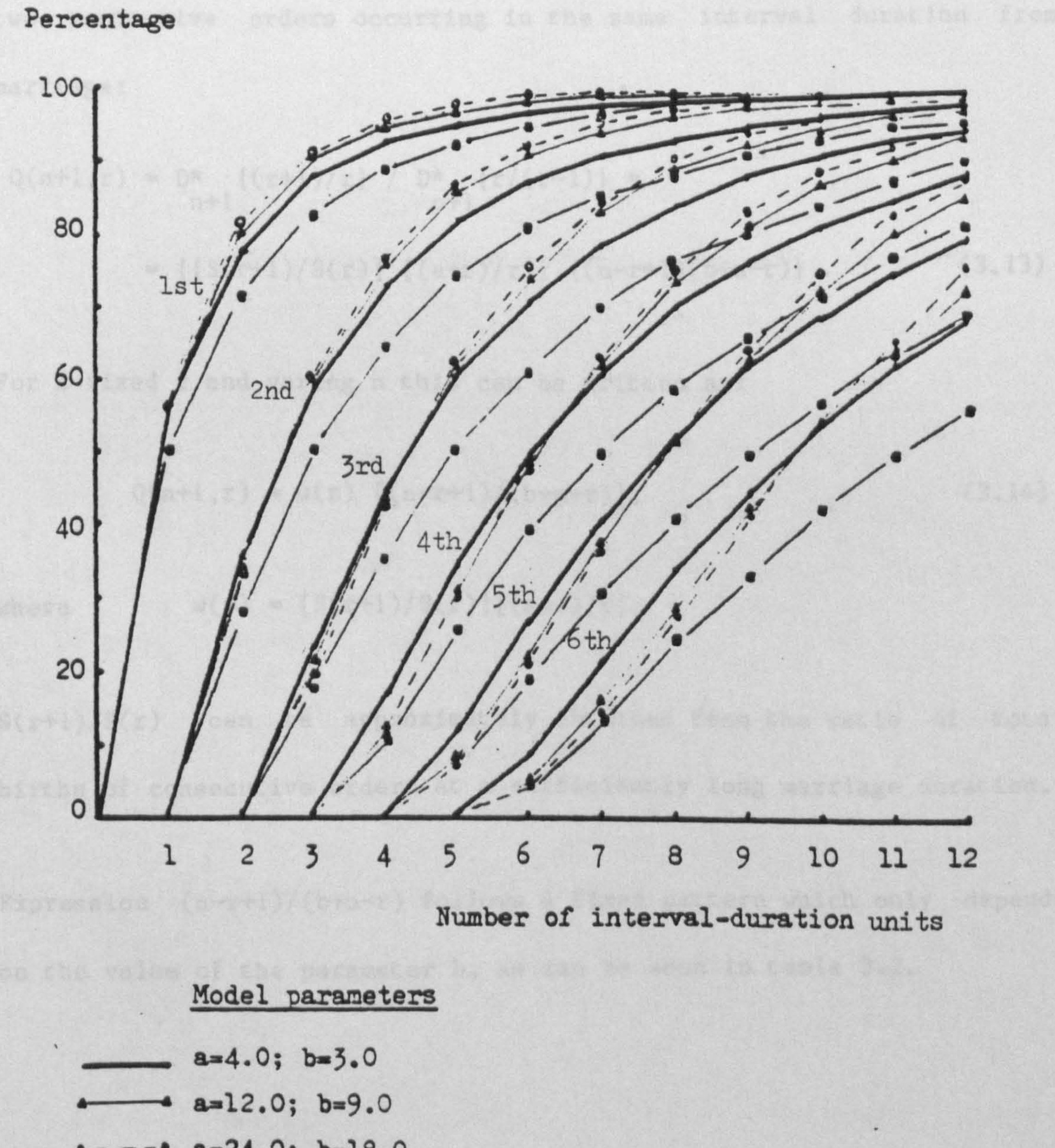
3.4 Applications of the fertility model to describing some fertility patterns observed in developing countries.

Following the work of other authors, the pace of fertility, given by the mean value of the women's fecundability ($\overline{p}$ = a/a+b), is determined by only one parameter, as the value a+b is assumed to be equal to 7. This is a very convenient simplification for practical purposes because, after fixing the variability of the distribution, only one parameter is left to be estimated. It does not greatly affect the results as the distribution is not very sensitive to changes in the variance (1/a+b), the dominant factor being the ratio a/(a+b).

This can be confirmed by observing figure 3.1, where the results from four models are compared; three of them have the same value $\overline{p}$=0.57, but greatly differing variabilities as a+b is 7, 21, and 42 respectively. The fourth model has a $\overline{p}$=0.50 and a+b=7. The first three models, with very different variances in the women's probabilities of having a birth in an interval (p) have cumulative distributions which are much closer than the fourth model is to the first one, which have the same variance, and not a big discrepancy in $\overline{p}$.

Figure 3.1

Cumulative per-cent distributions of births by order and duration of marriage from four models with different a and b parameters



The most efficient method for estimating the parameters of the beta distribution is the maximun likelihood method, derived by Griffiths (1973). For demographic applications of the same kind as those performed here Farahani (1981) and Brass (1982) have proposed simplified procedures. One of the estimation procedures proposed by Farahani is based on the use of the ratios of the number of births of two successive orders occurring in the same interval duration from marriage:

$$Q(n+1,r) = D^*_{n+1}\{(r+1)/r\} \;/\; D^*_{n+1}\{r/(r-1)\} =$$

$$= \{[S(r+1)/S(r)]\;[(a+r)/r]\}\;\{(n-r+1)(b+n-r)\} \qquad (3.13)$$

For a fixed r and varing n this can be written as:

$$Q(n+1,r) = w(r)\;[(n-r+1)/(b+n-r)]; \qquad (3.14)$$

where $\qquad w(r) = [S(r+1)/S(r)][(a+r)/r];$

$S(r+1)/S(r)$ can be approximately obtained from the ratio of total births of consecutive orders at a sufficiently long marriage duration.

Expression $(n-r+1)/(b+n-r)$ follows a fixed pattern which only depends on the value of the parameter b, as can be seen in table 3.2.

Table 3.2: Patterns of variation in expression $(n-r+1)/(b+n-r)$, by birth order (r) and interval-duration (n).

Intervals	Birth order (r)				
(n)	1	2	3	4	5
1	1 / b				
2	2/(b+1)	1 / b			
3	3/(b+2)	2/(b+1)	1 / b		
4	4/(b+3)	3/(b+2)	2/(b+1)	1 / b	
5	5/(b+4)	4/(b+3)	3/(b+2)	2/(b+1)	1 / b
6	6/(b+5)	5/(b+4)	4/(b+3)	3/(b+2)	2/(b+1)
... ...					

On this basis, writing $C=b-1$ and $K=n-r+1$, for any fixed r, $Q(n+1,r)$ can be re-written as:

$$Q(K,r)= w(r) \quad K/(C+K) \tag{3.15}$$

Expression (3.15) can be linearized as

$$Q(K,r) = w(r) - C\ Q(K,r) / K \ ; \qquad K=1,2, \ldots \tag{3.16}$$

w(r) and C in equation (3.16) can be estimated by mean squares, as the parameters which minimize the expression $Z = \sum_K (Q-Q^*)^2$. Writing y for Q(K,r) and x for Q(K,r)/K, $Z = \sum_K \{y_K - w(r) + C\ x\}^2$, which after differentiation with respect to w(r) and C gives the normal equations:

$$C = \{m \sum_K xy - \sum_K x \sum_K y\} \;/\; \{(\sum_K x)^2 - m \sum_K x^2\} \qquad (3.17)$$

$$w(r) = \{ \sum_K y + C \sum_K x \} \;/\; m \;; \qquad (3.18)$$

where m indicates the number of cases. This procedure can be useful when random variations are the main source of errors affecting these ratios. Very often in demographic analysis systematic errors can be more important that chance variations in causing departures from expected patterns. In the case of these Q ratios systematic errors can be very important. On these grounds Brass suggests the use of more rigid procedures to estimate the parameters of the distributions, obtaining a set of estimates for these parameters and selecting the most appropriate one on the basis of a demographic rather than a statistical criteria. With a+b fixed to the value of 7, only one parameter has to be estimated. An estimation for the pace of childbearing can be obtained from one of the four following ratios:

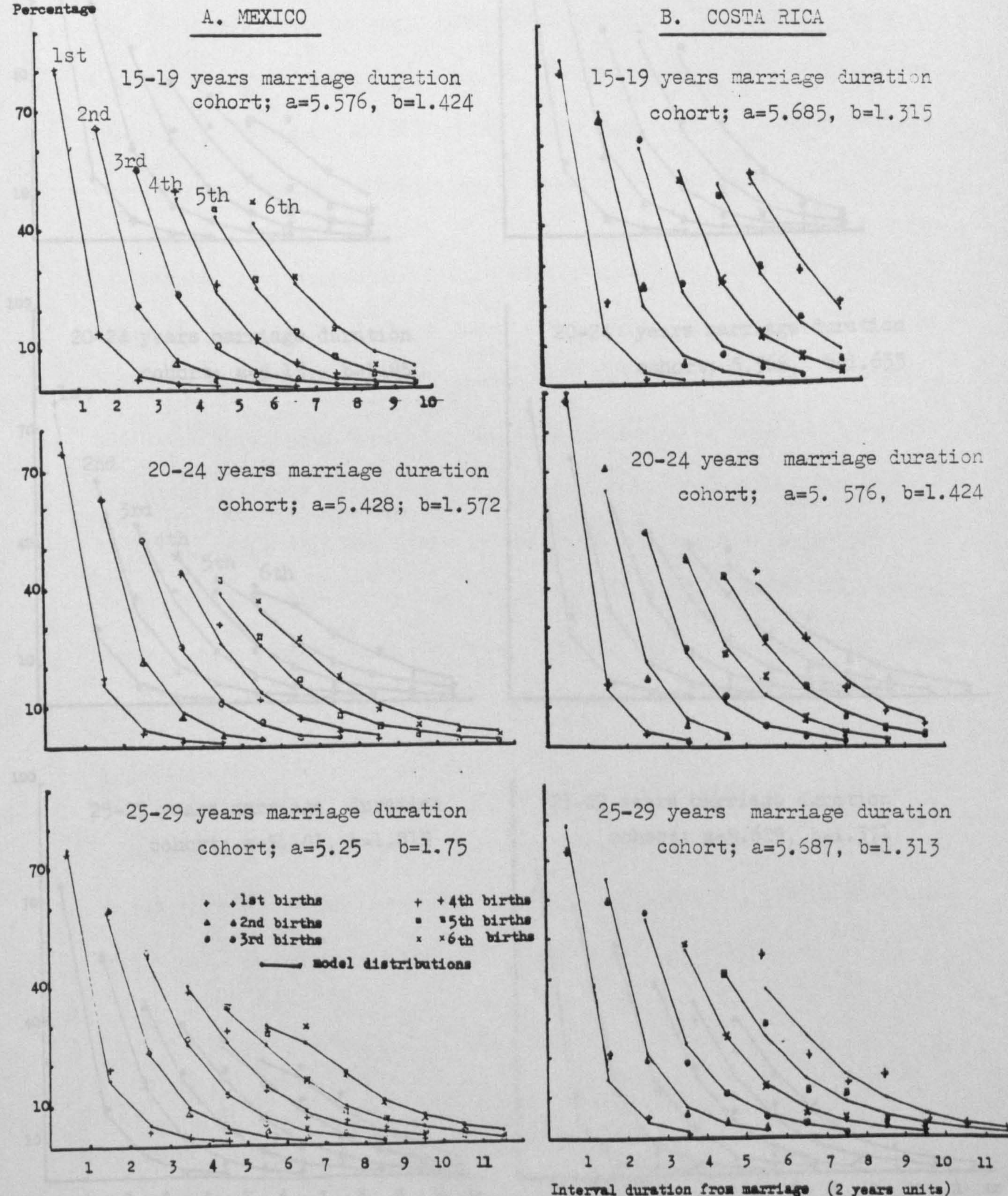
1) $R_1 = D^*_2\{1/0\} \;/\; D^*_1\{1/0\} = b \;/\; (a+b+1)$
2) $R_2 = D^*_3\{1/0\} \;/\; D^*_2\{1/0\} = (b+1) \;/\; (a+b+2)$
3) $R_3 = D^*_3\{2/1\} \;/\; D^*_2\{2/1\} = 2b \;/\; (a+b+2)$
4) $R_4 = D^*_4\{2/1\} \;/\; D^*_3\{2/1\} = 3b \;/\; 2(a+b+3)$

The first and the third ratios can be affected by variations in premaritally conceived births. As the pace estimate has tó be used for all births orderś, the movement from the first to the second birth appears to be a better basis for determining such pace since it is more central than the movement from marriage to first birth. On the basis of these considerations, Brass favours the fourth expression. However, the best choice can be dependent on the demographic characteristics of the population under study and the particular type of errors that may affect the data.

Figure 3.2 presents observed and fitted model distributions of births by order and duration of marriage for some countries. For each country three marriage-duration-cohorts are analysed. Clear structures by birth order and marriage duration appear, and the patterns underlying these structures are closely described by the model. The agreement between the observed and the fitted model distributions is very good.

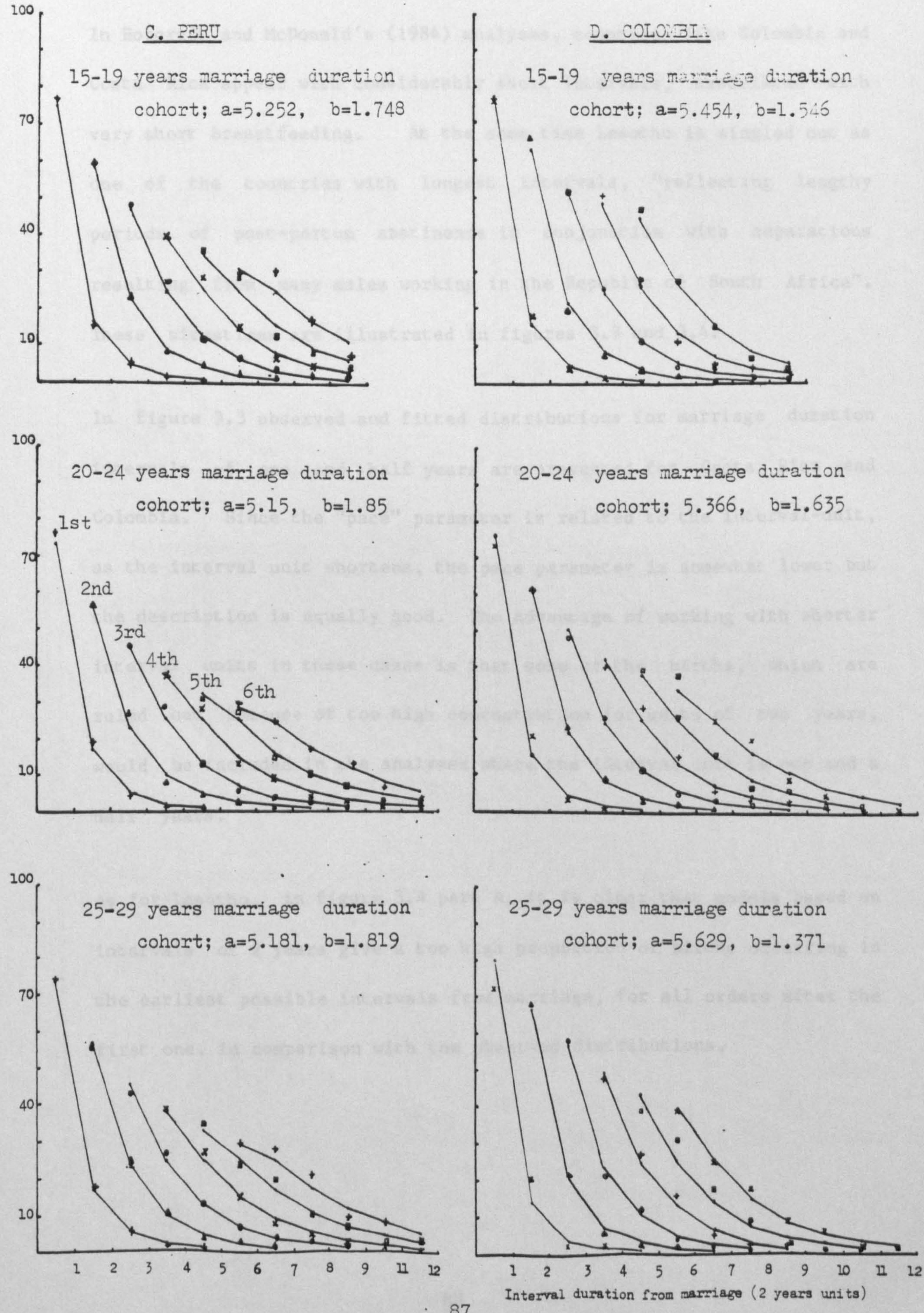
The observed data do not show any systematic departure from the model distributions. This picture reinforces the conclusions drawn by Brass, Farahani, Pellizi and Penhale in previous analyses, that there is an underlying common structure to distributions by birth order and duration of marriage, and the model can be used to characterize such structure in terms of a few parameters.

Figure 3.2: Observed and model distributions of births by order and marriage duration for some selected countries.



(Continue...)

Figure 3.2 (Continuation)



In Hobcraft and McDonald's (1984) analyses, countries like Colombia and Costa Rica appear with considerably short intervals, associated with very short breastfeeding. At the same time Lesotho is singled out as one of the countries with longest intervals, "reflecting lengthy periods of post-partum abstinence in conjunction with separations resulting from many males working in the Republic of South Africa". These situations are illustrated in figures 3.3 and 3.4.

In figure 3.3 observed and fitted distributions for marriage duration intervals of one and half years are presented for Costa Rica and Colombia. Since the "pace" parameter is related to the interval-unit, as the interval unit shortens, the pace parameter is somewhat lower but the description is equally good. The advantage of working with shorter interval units in these cases is that some of the births, which are ruled out because of too high concentration for units of two years, would be included in the analyses where the interval unit is one and a half years.

As for Lesotho, in figure 3.4 part A, it is clear that models based on intervals of 2 years give a too high proportion of births occurring in the earliest possible intervals from marriage, for all orders after the first one, in comparison with the observed distributions.

Figure 3.3: Observed and model distributions of births by 1½ years intervals from marriage for Costa Rica and Colombia.

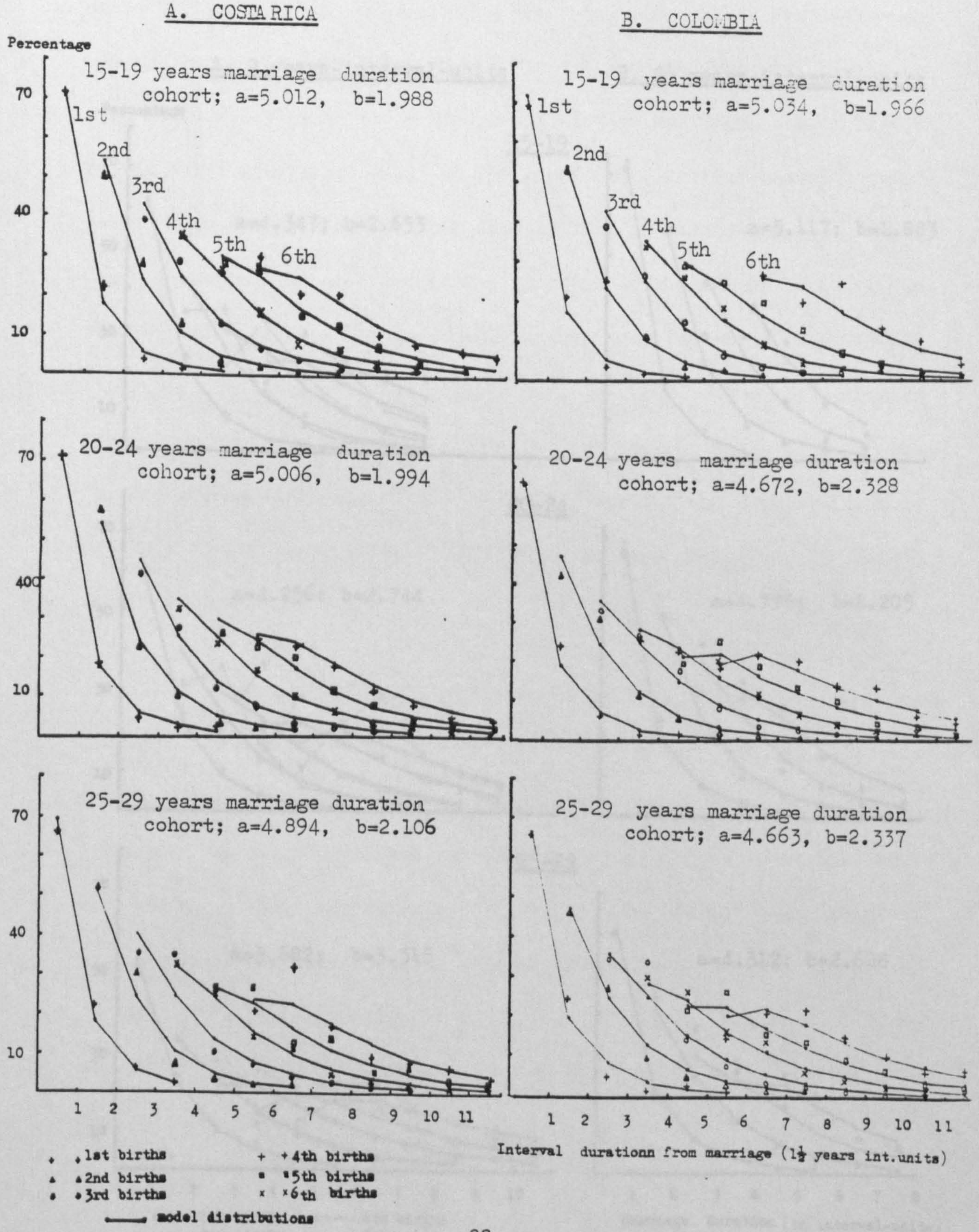
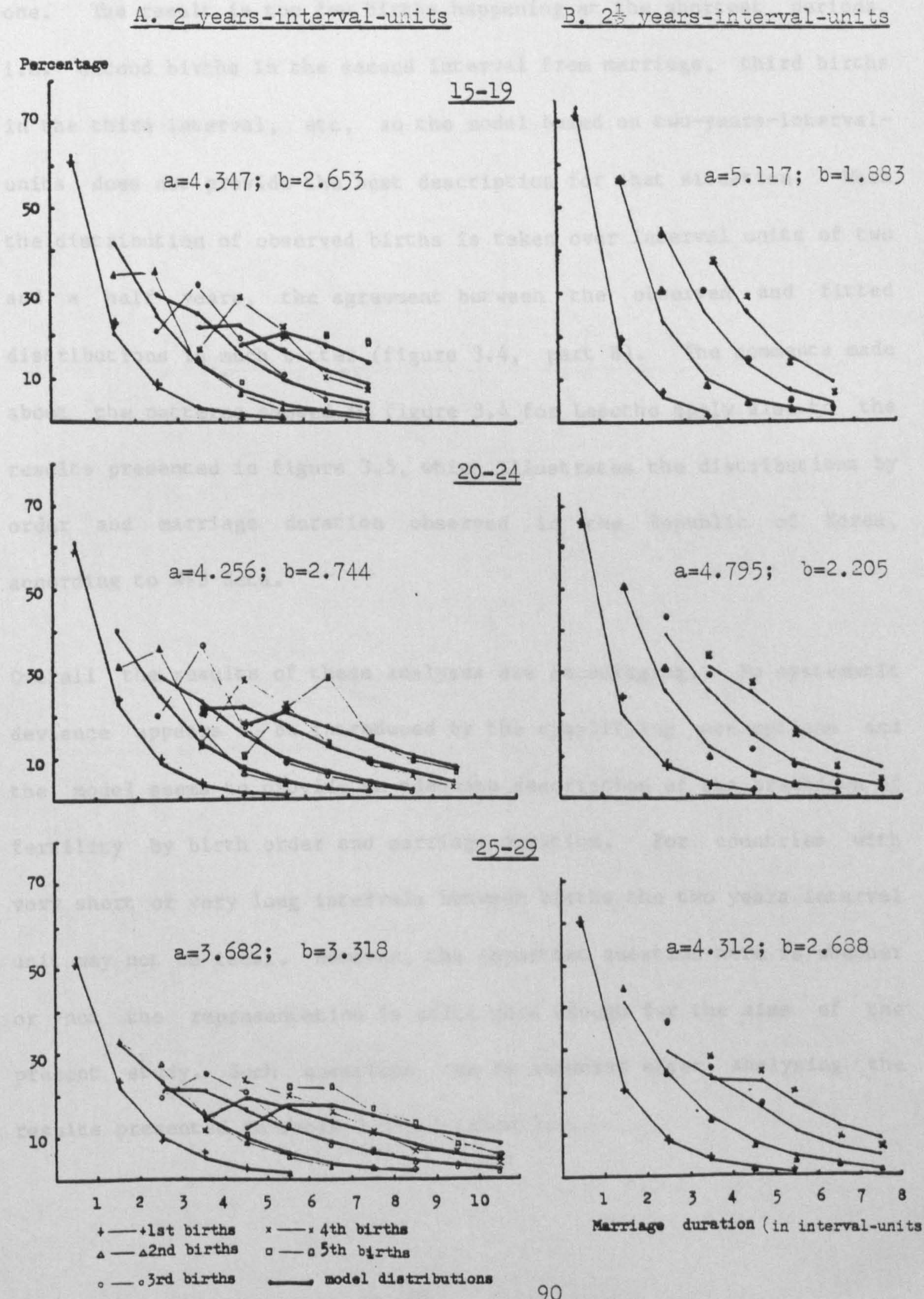


Figure 3.4: Observed and model distributions of births by 2 and $2\frac{1}{2}$ years-intervals-marriage duration for Lesotho.



As this interval unit (24 months) is too short (for the particular case of Lesotho) the probability of a birth in an interval is not entirely independent on whether a birth has occurred in the previous one. The result is too few births happening at the shortest periods, i.e. second births in the second interval from marriage, third births in the third interval, etc, so the model based on two-years-interval-units does not provide the best description for that situation. When the distribution of observed births is taken over interval units of two and a half years, the agreement between the observed and fitted distributions is much better (figure 3.4, part B). The comments made about the patterns showed in figure 3.4 for Lesotho apply also to the results presented in figure 3.5, which illustrates the distributions by order and marriage duration observed in the Republic of Korea, according to WFS data.

Overall the results of these analyses are encouraging. No systematic deviance appears to be introduced by the symplifying assumptions and the model seems to provide an adequate description of the breakdown of fertility by birth order and marriage duration. For countries with very short or very long intervals between births the two years interval unit may not be ideal. However, the important question here is whether or not the representation is still good enough for the aims of the present study. Such questions can be answered after analysing the results presented in table 3.3 and table 3.4.

Figure 3.5; Observed and model distributions of births by two and two and a half years-interval-marriage duration for the Republic of Korea

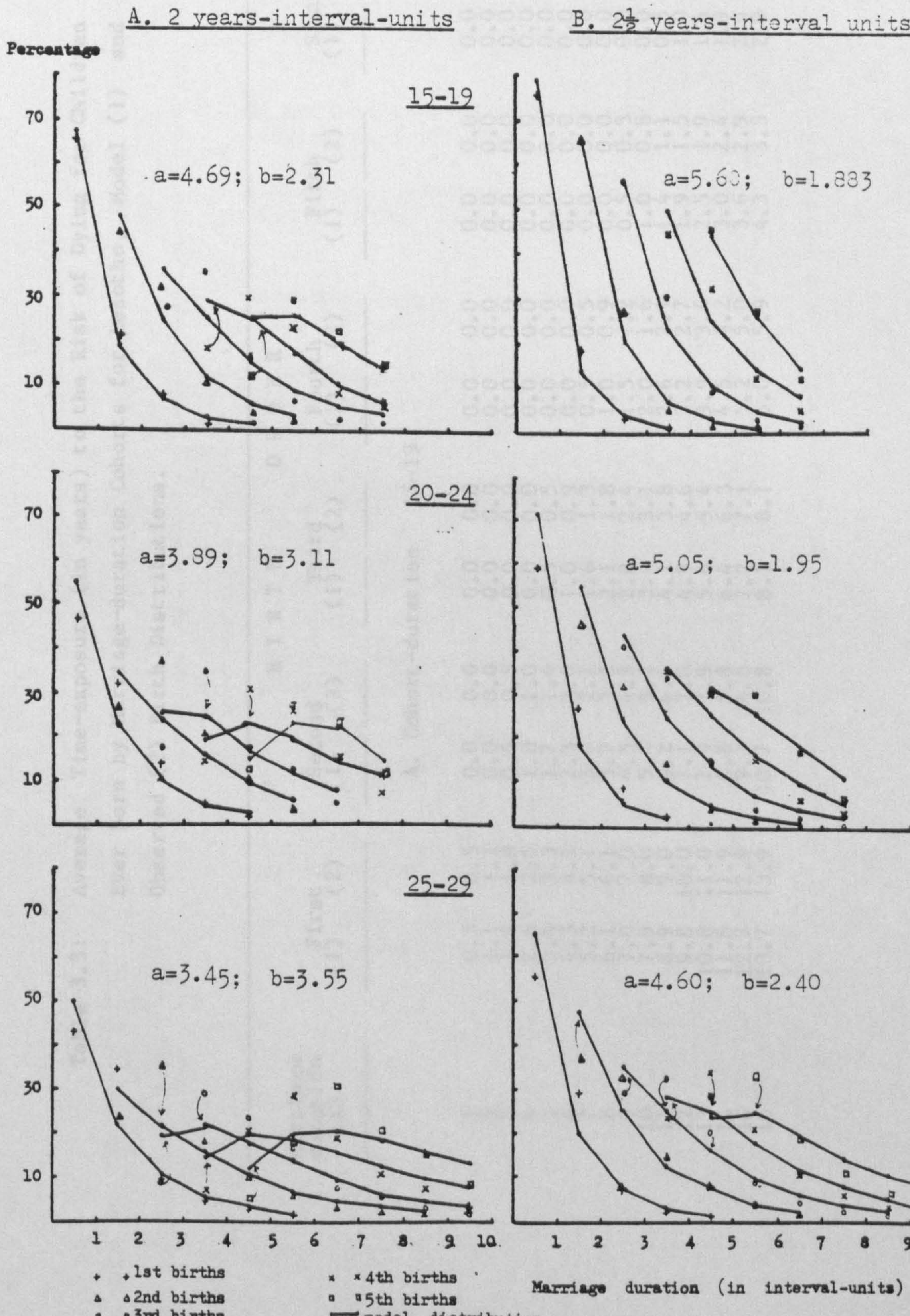


Table 3.3: Average Time-exposure (in years) to the Risk of Dying for Children Ever Born by Marriage-duration Cohorts for Lesotho, Model (1) and Observed (2) Birth Distributions.

	BIRTH ORDER											
Marriage duration (i)	First (1)	First (2)	Second (1)	Second (2)	Third (1)	Third (2)	Fourth (1)	Fourth (2)	Fifth (1)	Fifth (2)	Sixth (1)	Sixth (2)
	A. Cohort-duration 15-19											
1	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	1.1	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	1.8	1.8	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	2.6	2.5	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	3.4	3.3	1.7	1.4	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0
6	4.3	4.2	2.3	2.0	1.0	0.9	0.0	0.0	0.0	0.0	0.0	0.0
7	5.2	5.1	3.0	2.7	1.6	1.3	0.5	0.5	0.0	0.0	0.0	0.0
8	6.1	6.1	3.7	3.5	2.1	1.8	1.0	0.9	0.0	0.0	0.0	0.0
9	7.0	7.0	4.5	4.3	2.8	2.4	1.5	1.2	0.5	0.5	0.0	0.0
10	7.9	8.0	5.4	5.2	3.4	3.1	2.0	1.6	1.0	0.8	0.0	0.0
11	8.9	9.0	6.2	6.1	4.1	3.8	2.6	2.1	1.4	1.1	0.5	0.5
12	9.8	10.0	7.1	7.0	4.9	4.6	3.2	2.7	1.9	1.5	1.0	0.9
13	10.8	11.0	7.9	7.9	5.6	5.4	3.9	3.4	2.5	1.9	1.4	1.3
14	11.8	11.9	8.8	8.8	6.4	6.3	4.5	4.2	3.0	2.4	1.9	1.7
15	12.7	12.9	9.7	9.8	7.2	7.1	5.2	5.0	3.6	2.9	2.4	2.0
16	13.7	13.9	10.7	10.8	8.1	8.1	6.0	5.9	4.3	3.5	2.9	2.4

Table 3.3 (continuation)

Marriage duration (1)	First (1)	First (2)	Second (1)	Second (2)	Third (1)	Third (2)	Fourth (1)	Fourth (2)	Fifth (1)	Fifth (2)	Sixth (1)	Sixth (2)
	B. Cohort-duration 20-24											
1	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	1.1	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	1.8	1.8	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	2.6	2.5	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	3.4	3.3	1.6	1.4	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0
6	4.3	4.2	2.3	1.9	1.0	0.9	0.0	0.0	0.0	0.0	0.0	0.0
7	5.1	5.0	3.0	2.6	1.6	1.2	0.5	0.5	0.0	0.0	0.0	0.0
8	6.0	6.0	3.7	3.4	2.1	1.6	1.0	0.9	0.0	0.0	0.0	0.0
9	6.9	6.9	4.5	4.2	2.7	2.3	1.5	1.2	0.5	0.5	0.0	0.0
10	7.8	7.8	5.3	5.1	3.4	3.0	2.0	1.6	1.0	0.9	0.0	0.0
11	8.8	8.8	6.1	6.0	4.1	3.8	2.6	2.2	1.4	1.3	0.5	0.5
12	9.8	9.8	7.0	6.9	4.8	4.6	3.2	2.7	1.9	1.7	1.0	0.9
13	10.7	10.7	7.8	7.7	5.6	5.4	3.8	3.4	2.4	2.0	1.4	1.3
14	11.7	11.7	8.7	8.6	6.3	6.2	4.5	4.1	3.0	2.4	1.9	1.7
15	12.7	12.7	9.6	9.6	7.1	7.0	5.2	4.9	3.6	3.0	2.4	2.0
16	13.6	13.7	10.5	10.6	8.0	7.9	5.9	5.6	4.2	3.8	2.9	2.3
17	14.6	14.7	11.5	11.5	8.8	8.8	6.6	6.4	4.9	4.5	3.4	2.8
18	15.6	15.7	12.4	12.5	9.7	9.7	7.4	7.3	5.5	5.3	4.0	3.4
19	16.6	16.7	13.3	13.5	10.5	10.7	8.2	8.2	6.2	6.1	4.6	4.1
20	17.6	17.7	14.3	14.5	11.4	11.7	9.0	9.1	6.9	7.0	5.2	4.9

(1) Average exposures obtained from the model distribution with a=4.347 and b=2.653 for the cohort-duration 15-19, and a=4.256, b=2.744 for the cohort-duration 20-24 year.

(2) Average exposures obtained from the observed distributions of birth by order and duration.

Table 3.4: Average Exposure to the Risk of Dying (in years) for Children Ever Born by Order and Age of the Mothers, Lesotho, Calculated from Model (1) and Observed (2) Birth Distributions.

Mother's age (1)	BIRTH ORDER First (1)	First (2)	Second (1)	Second (2)	Third (1)	Third (2)	Fourth (1)	Fourth (2)	Fifth (1)	Fifth (2)	Sixth (1)	Sixth (2)
(*)	A. Calculated from the distribution of births by duration from duration-cohort 15-19											
1	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.6	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.8	0.8	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	1.0	1.0	0.6	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	1.2	1.2	0.8	0.8	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0
6	1.4	1.4	0.9	0.9	0.6	0.6	0.0	0.0	0.0	0.0	0.0	0.0
7	1.7	1.7	1.1	1.1	0.8	0.8	0.5	0.5	0.0	0.0	0.0	0.0
8	2.0	2.0	1.4	1.3	0.9	0.9	0.6	0.6	0.0	0.0	0.0	0.0
9	2.3	2.3	1.6	1.5	1.1	1.1	0.8	0.7	0.5	0.5	0.0	0.0
10	2.7	2.7	1.9	1.8	1.3	1.3	0.9	0.9	0.6	0.6	0.0	0.0
11	3.2	3.2	2.2	2.1	1.6	1.5	1.1	1.0	0.8	0.7	0.5	0.5
12	3.7	3.7	2.6	2.5	1.8	1.7	1.3	1.2	0.9	0.9	0.6	0.6
13	4.3	4.2	3.0	2.9	2.2	2.0	1.5	1.4	1.1	1.0	0.8	0.8
14	4.9	4.9	3.5	3.3	2.5	2.3	1.8	1.7	1.3	1.2	0.9	0.9
15	5.6	5.6	4.0	3.8	2.9	2.7	2.1	1.9	1.5	1.4	1.1	1.1
16	6.3	6.3	4.5	4.4	3.3	3.1	2.4	2.2	1.8	1.6	1.3	1.2
17	7.1	7.1	5.1	5.0	3.8	3.5	2.8	2.5	2.0	1.8	1.5	1.4
18	7.9	7.9	5.8	5.6	4.3	4.0	3.2	2.9	2.4	2.1	1.7	1.6
19	8.7	8.8	6.5	6.4	4.8	4.6	3.6	3.3	2.7	2.4	2.0	1.9
20	9.6	9.7	7.2	7.1	5.4	5.2	4.1	3.8	3.1	2.7	2.3	2.1
21	10.5	10.6	8.0	7.9	6.0	5.8	4.6	4.3	3.5	3.1	2.6	2.4
22	11.4	11.6	8.8	8.8	6.7	6.5	5.1	4.9	3.9	3.5	3.0	2.7
23	12.4	12.5	9.6	9.6	7.4	7.3	5.7	5.5	4.4	4.0	3.4	3.1
24	13.3	13.5	10.5	10.5	8.1	8.1	6.3	6.2	4.9	4.5	3.8	3.5
25	14.3	14.5	11.3	11.4	8.9	8.9	7.0	6.9	5.4	5.1	4.2	3.9

Table 3.4 (continuation)

Mother's age (i)	BIRTH ORDER First (1)	First (2)	Second (1)	Second (2)	Third (1)	Third (2)	Fourth (1)	Fourth (2)	Fifth (1)	Fifth (2)	Sixth (1)	Sixth (2)
(*)	B. Calculated from the distribution of births by duration from cohort-duration 20-24											
1	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.6	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.8	0.8	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	1.0	1.0	0.6	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	1.2	1.2	0.8	0.8	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0
6	1.4	1.4	0.9	0.9	0.6	0.6	0.0	0.0	0.0	0.0	0.0	0.0
7	1.7	1.7	1.1	1.1	0.8	0.7	0.5	0.5	0.0	0.0	0.0	0.0
8	2.0	2.0	1.4	1.3	0.9	0.9	0.6	0.6	0.0	0.0	0.0	0.0
9	2.3	2.3	1.6	1.5	1.1	1.0	0.8	0.7	0.5	0.5	0.0	0.0
10	2.7	2.7	1.9	1.8	1.3	1.2	0.9	0.9	0.6	0.6	0.0	0.0
11	3.2	3.2	2.2	2.1	1.6	1.4	1.1	1.0	0.8	0.8	0.5	0.5
12	3.7	3.7	2.6	2.4	1.8	1.7	1.3	1.2	0.9	0.9	0.6	0.6
13	4.3	4.2	3.0	2.8	2.1	1.9	1.5	1.4	1.1	1.1	0.8	0.8
14	4.9	4.8	3.4	3.3	2.5	2.3	1.8	1.6	1.3	1.2	0.9	0.9
15	5.5	5.5	3.9	3.7	2.9	2.6	2.1	1.9	1.5	1.4	1.1	1.1
16	6.3	6.2	4.5	4.3	3.3	3.0	2.4	2.2	1.8	1.6	1.3	1.2
17	7.0	7.0	5.1	4.9	3.7	3.5	2.8	2.5	2.0	1.9	1.5	1.4
18	7.8	7.8	5.7	5.5	4.2	4.0	3.1	2.9	2.3	2.1	1.7	1.6
19	8.7	8.7	6.4	6.2	4.8	4.5	3.6	3.3	2.7	2.4	2.0	1.9
20	9.6	9.5	7.1	7.0	5.3	5.1	4.0	3.8	3.0	2.8	2.3	2.1
21	10.5	10.5	7.9	7.8	6.0	5.8	4.5	4.3	3.4	3.2	2.6	2.4
22	11.4	11.4	8.7	8.6	6.6	6.5	5.1	4.8	3.9	3.6	3.0	2.7
23	12.3	12.3	9.5	9.5	7.3	7.2	5.6	5.4	4.3	4.1	3.3	3.0
24	13.3	13.3	10.4	10.3	8.1	8.0	6.2	6.1	4.8	4.6	3.8	3.4
25	14.2	14.3	11.2	11.3	8.8	8.8	6.9	6.8	5.4	5.2	4.2	3.9

(1) Obtained by combining the model distribution, truncated at the appropriate duration with a nuptiality model with mean age at 19.7 and variance 11.0 (g=0.68 and h=17.0 in the negative binomial distribution, the nuptiality model described in the following chapter).

(2) Obtained from the observed distribution by duration and the nuptiality model used in (1).

(*) Age (i) from an arbitrary origin at the onset of nuptiality (about 11 for Lesotho).

The mean time-exposure to the risk of dying, for children born to women in the different cohorts, are the basis of all the calculations necessary for this study. Therefore, such measures provide a sensible indicator for evaluating the importance of the biases that may arise if a unique lenght of two years for the interval-unit is applied when describing fertility patterns for all countries.

The average time-exposures obtained from the distributions observed in Lesotho (WFS data, marriage cohorts 15-19 years duration and 20-24 years duration) are compared with the exposures calculated from the fitted distributions. These two cohorts were selected for the analyses because, as it can be seen in figure 3.4, they are the cases in which the two-years-interval-unit-model gave the poorest description. In any other case the bias would be smaller. The way the average exposures were obtained is explained in Chapter 5.

The results presented in table 3.3 indicate that the bias introduced by the imperfect description is not serious. For each birth order and marriage duration the average time-exposures obtained from the model distribution (1) are compared with those obtained from the observed distribution (2). The biggest differences appear in the case of fourth births, for durations of between 11 and 13 years, reaching half a year. These differences are minimized still further, as the duration model is later combined with a nuptiality model to obtain distributions by age of the mother, and these are the results relevant to the calculation procedure which interests us, as will be seen in later chapters. The effects on the estimated average exposures by age of the mother and birth order are presented in table 3.4. The same nuptiality distribution (a model with mean at age 19.7 and variance equal to 11, which describes closely the nuptiality patterns in Lesotho, as will be seen in Chapter 4) was applied to the observed and the fitted distributions by order and duration. There is no doubt that, for the purposes of this study, the approximation is quite good. The biggest difference is 0.3 years, and that for the cohort and country where the model showed the poorest performance. Further refinements, considering different interval-unit lenghts, do not seem to be justified at this stage in the light of the considerable additional calculations that it would demand, and taking into account that all we need is a reasonable approximation. Furthermore, several other simplifications will have to be introduced later in the calculation process anyway.

As was pointed out above, in order to obtain distributions of births by order and age of the mother, a nuptiality model is required for describing the distribution of age at marriage. The fertility distribution by age of the mother and birth order is subsequently found by combining the distribution of ages at marriage with the fertility model by duration of marriage. In the next chapter the nuptiality model is discussed and then the fertility model by birth order and age of the women is introduced.

CHAPTER 4

The Nuptiality Model and the Model of Fertility by Age and Birth Order.

IV. THE NUPTIALITY MODEL AND THE MODEL OF FERTILITY BY AGE AND BIRTH ORDER.

4.1 Introduction

Among the demographic models for describing distributions of age at first marriage, the one proposed by Coale (1971) has probably been the one most widely used. Coale's nuptiality model is based on the empirical observation that, even for widely differing types of societies, the distributions of age at marriage for ever married women have the same basic form. Indeed the agreement in such distributions is remarkable once they have been standardized by linear transformations in the age scale and the final proportion of women eventually marrying in each cohort. Coale represented the standard form on the basis of period data from Sweden in the last century (1865-1869). The model was developed further by Coale and McNeil (1972), replacing the standard empirical schedule by a mathematical expression. Hence, the distribution of ages at first marriage, g(a), is described as:

$$g(a) = C \;\; 0.19465/K \exp\{[-0.174(a-a_0-6.06K)/K] - \exp[-0.2881(a-a_0-6.06K)/K]\} \qquad (4.1)$$

where a_0 represents an age at which a significant number of first marriages occur; K is a scale parameter representing the pace of nuptiality in the cohort, determined by the ratio between the number of years in the time span during which the first marriages occur in the observed population and that number in the standard population; C is the proportion of ever married women in the cohort. Rodriguez and Trussell (1980) reformulated the model in terms of the mean and the standard deviation of the distribution and provided a maximum likelihood estimation procedure to fit the model to survey data.

Coale's model was used to represent the nuptiality component in the Coale and Trussell (1974) model fertility schedules, and in many other procedures where an expression for the distribution of ages at first marriage was required. The model is based in a continuous function and for some type of calculations it is not easy to handle. Considering the requirements of the present study, the model introduced by Farahani (1981), which consists on a negative binomial distribution, was preferred.

Although the negative binomial model has been used in demographic applications as early as 1957 (Brass,1957), it has not became very popular among demographers. Previous demographic applications of the negative binomial distribution have been mainly for describing distributions of women by completed family size (Brass 1958.a, Brass 1958.b). For more details about the negative binomial distribution see, for example, Moran (1968).

4.2 The negative binomial distribution as a nuptiality model

Following some ideas from Feeney (as quoted by Farahani) "that the marriage curve may be composed of a random age of entry followed by a random delay", Farahani represented the distribution of entry into the marriage market as a negative binomial, and the distribution of delay as a simple geometric with the same ratio parameters. On such assumptions he found that the age interval at first marriage follows a negative binomial distribution:

$$M(x) = [(h+x-1)! \,/\, h!\,(x-1)!]\ g^{h+1}\,(1-g)^{x-1} \quad ; \quad x = 1,2,\ \ldots \qquad (4.2)$$

where x represents the age intervals from an arbitrary starting point of the nuptiality process. To refer to a specific population, another parameter, representing the age at the start of nuptiality (equivalent to a_0 in Coale's model), is necessary.

- g and h are parameters which characterize the negative binomial distribution; $0<g<1$, while the only restriction for h is that it must be positive.

The mean and the variance of this distribution are:

$$\mu = 1 + (h+1)\,(1-g) \,/\, g \qquad (4.3)$$

$$\sigma^2 = (h+1)\,(1-g) \,/\, g^2 \qquad (4.4)$$

For a given value of h, the higher the value of g, the more concentrated the distribution will be on the first intervals. Thus, when g is higher (closer to 1) the mean will be lower, and so will be the variance. On the other hand, for a fixed g value, the distribution of nuptiality will have a larger spread and the mean will be higher as h increases. If both parameters are modified the final effect on the distribution depends on the combined effect. It may concentrate or spread the distribution, increase or decrease the mean, according to the degree of change in each of the two parameters.

Evaluation of expression 4.2 is very easy taking into account that:

$$M(1) = g^{h+1} \tag{4.5}$$

$$M(x+1) = (h+x)/x\ (1-g)\ M(x) \tag{4.6}$$

Since h and g are known parameters of the distribution, M(1) can be calculated and the probabilities for all the following intervals can be obtained from equation 4.6

The negative binomial representation has the advantage of being a simple, closed form frequency function. For our purposes here, it provides a neat and easy way to handle discrete representation, which can be combined with the beta binomial distribution, analysed in the previous chapter, to obtain a distribution of births by order and age of the mother. As indicated above, the distribution of interval-ages at first marriage is given from an arbitrary origin at which the women begin to enter the marriage market. To express such a distribution in terms of completed years of age in a given population, this origin has to be specified. It is also necessary to take into account that the parameter values from equations 4.3 and 4.4 correspond to a function of discrete variable. The mean, as determined by 4.3, would imply that the marriages occur at the end of each interval. This value should be adjusted if it is assumed that marriages occur at the begining or at mid point of the interval. In order to get the feeling of the model and of the effects on the shape of the distribution caused by variations in the values of its parameters, in the next section the model is fitted to real data, obtained from W.F.S. surveys. At the same time the exercise provides a test of the flexibility of the model for describing different nuptiality patterns.

4.3 Fitting the negative binomial distribution to survey data.

For methods of fitting the negative binomial distribution Fisher (1941), Anscombe (1950), and Williamson and Bretherton (1963), can be consulted. As we are not concerned in this particular study with the best fitting, a reasonable approximation will be sufficient for our purposes. Hence, the parameters h and g are obtained by equating the sample estimates for the mean and the variance to the population parameters in equations (4.3) and (4.4), and solving the system for h and g. This is not the most efficient method of fitting the negative binomial, but it is very simple and provides good enough results for our purposes.

The model was fitted to marriage histories from fertility surveys conducted within the W.F.S. programme in the following countries: Colombia, Costa Rica, Lesotho, Mexico, Peru, and the Republic of Korea. For each country information from four age-cohorts of women was analysed. In all cases the marriage distributions were truncated at the age of 35; as very few marriages were recorded after that age, this has little effect on the model parameters.

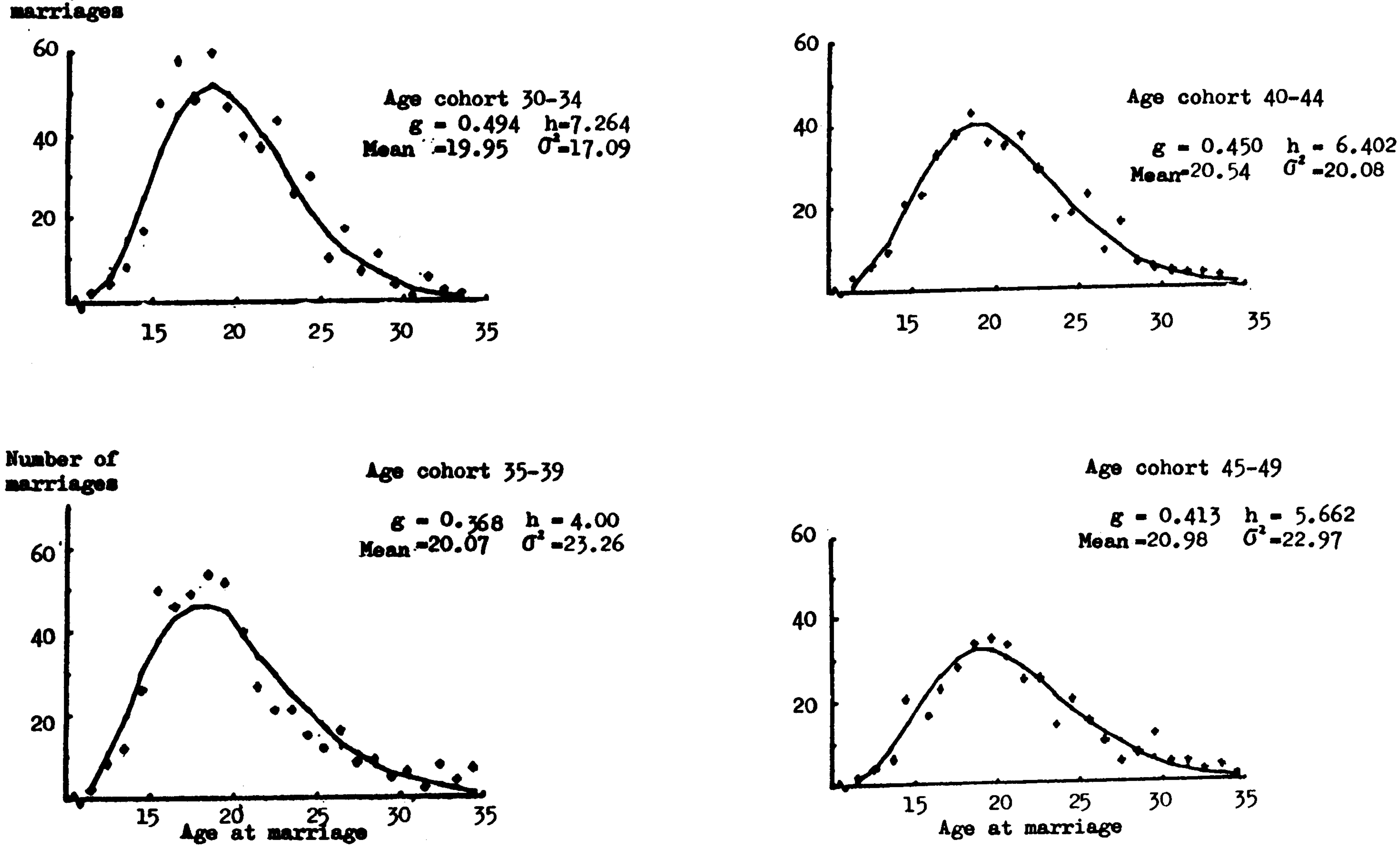
The results are presented graphically in figure 4.1. The model fits the data very well. There is no doubt that, for the simplified representation which is needed in this study, the negative binomial gives and exceedingly good description of the nuptiality processes that are observed in most countries. The model describes satisfactorily experiences that range from that of Sweden 1865-1869 (Coale's standard) where marriages occur through a time span of about 40 years (fitted in Farahani,1981, page 165), with a SMAM value of about 11 years from the onset of nuptiality and a variance of 34, to that of Korean women (WFS data), age cohort 45-49 years, where all women married within a time span of fourteen years with the mean of the distribution at about 5 years from the origin, and a variance of 5.

Although this is not within the concerns of this investigation, it is interesting to note that changes in nuptiality in a given country, such as those which took place in the Republic of Korea from one age cohort to another, are well described by the model, and that this model distribution can provide a useful and manageable tool for analysing the characteristics of such changes. In Korea a massive change took place between the cohort aged 45-49, and the cohort aged 30-34 at the survey, which resulted in later ages at first marriage and a more widely spread distribution in the younger cohort. Such changes are brougt out when the four cohorts are super-imposed in the same graph, as in figure 4.2.

Although not on such a big scale as those in the Republic of Korea, changes in Colombia are also significant, and in an unexpected direction: cohort 30-34 presents a mean age at first marriage one year younger than cohort 45-49. In Latin-American countries, where cohabitation frequently begins some time before the formal marriage ceremony, a tendency in older women to report the date of the formal marriage as the start of the union, perhaps together with some changes in social practices (formalizing unions earlier), may produce such apparent changes in the marriage distribution without any significant change in the time exposure to fertility.

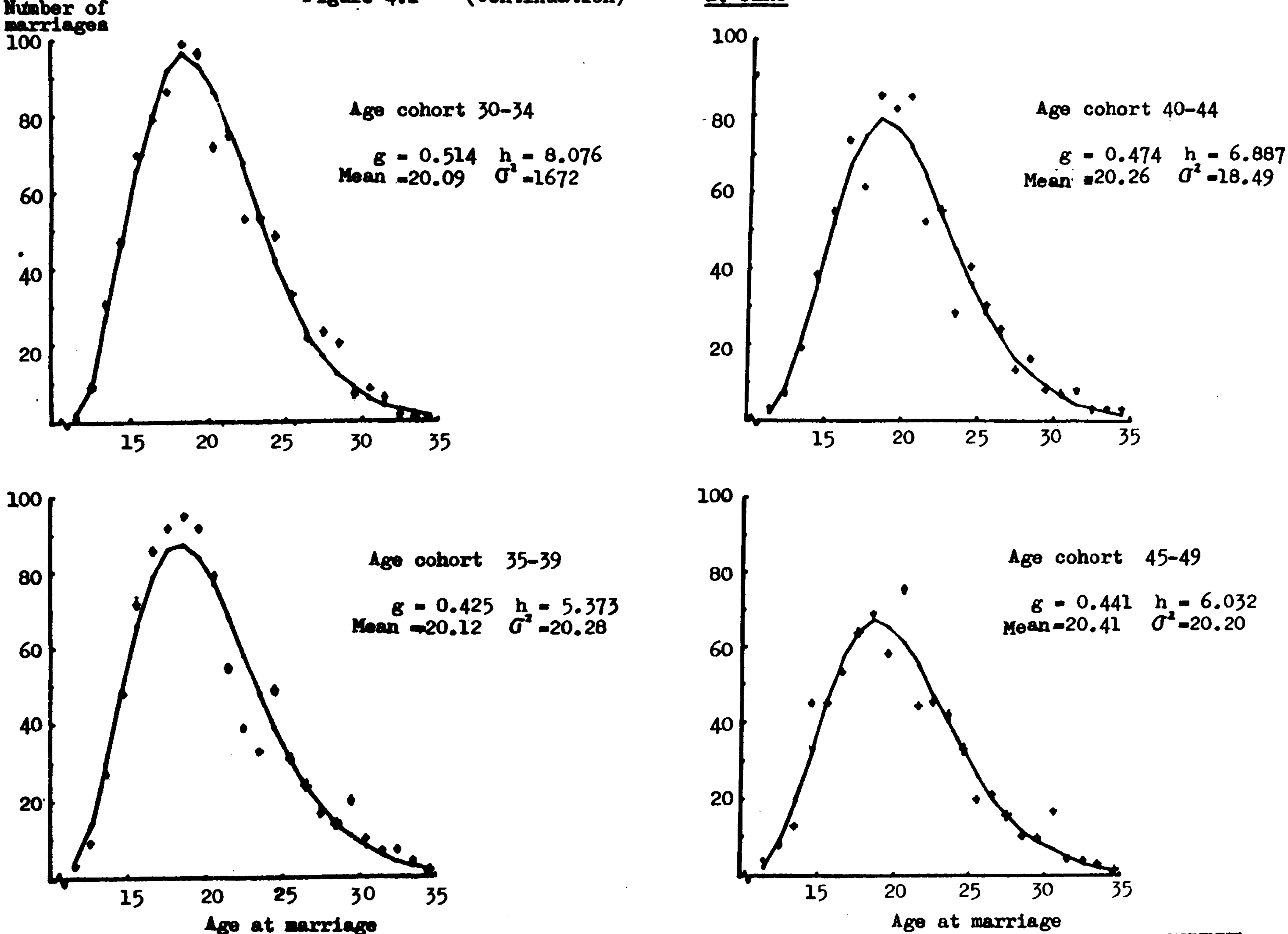
Figure 4.1 Observed and expected numbers of first marriages by age of the women for selected age-group-cohorts and countries.-

A. COLOMBIA



CONTINUE...

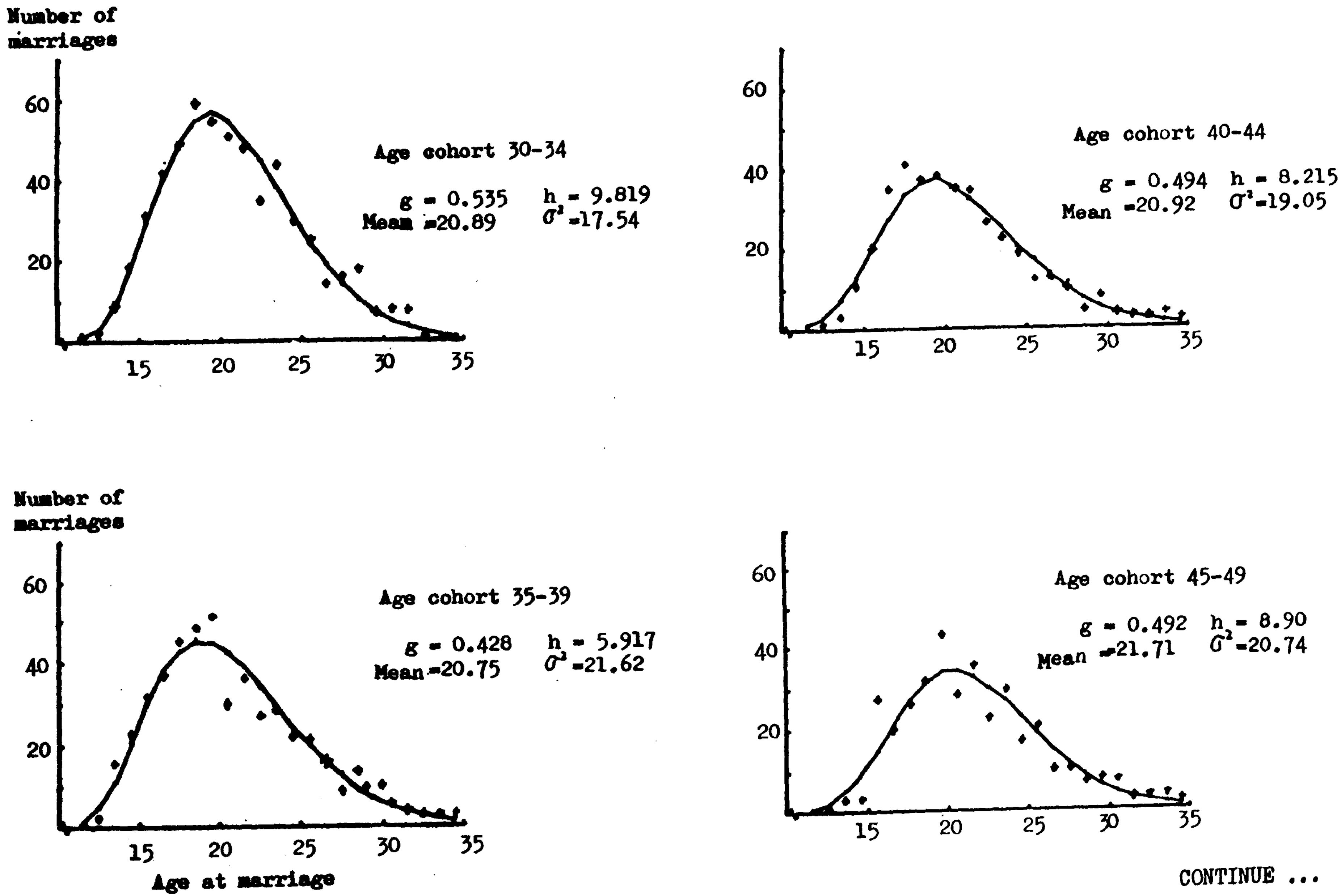
Figure 4.1 (Continuation) B. PERU



CONTINUE ...

Figure 4.1 (Continuation)

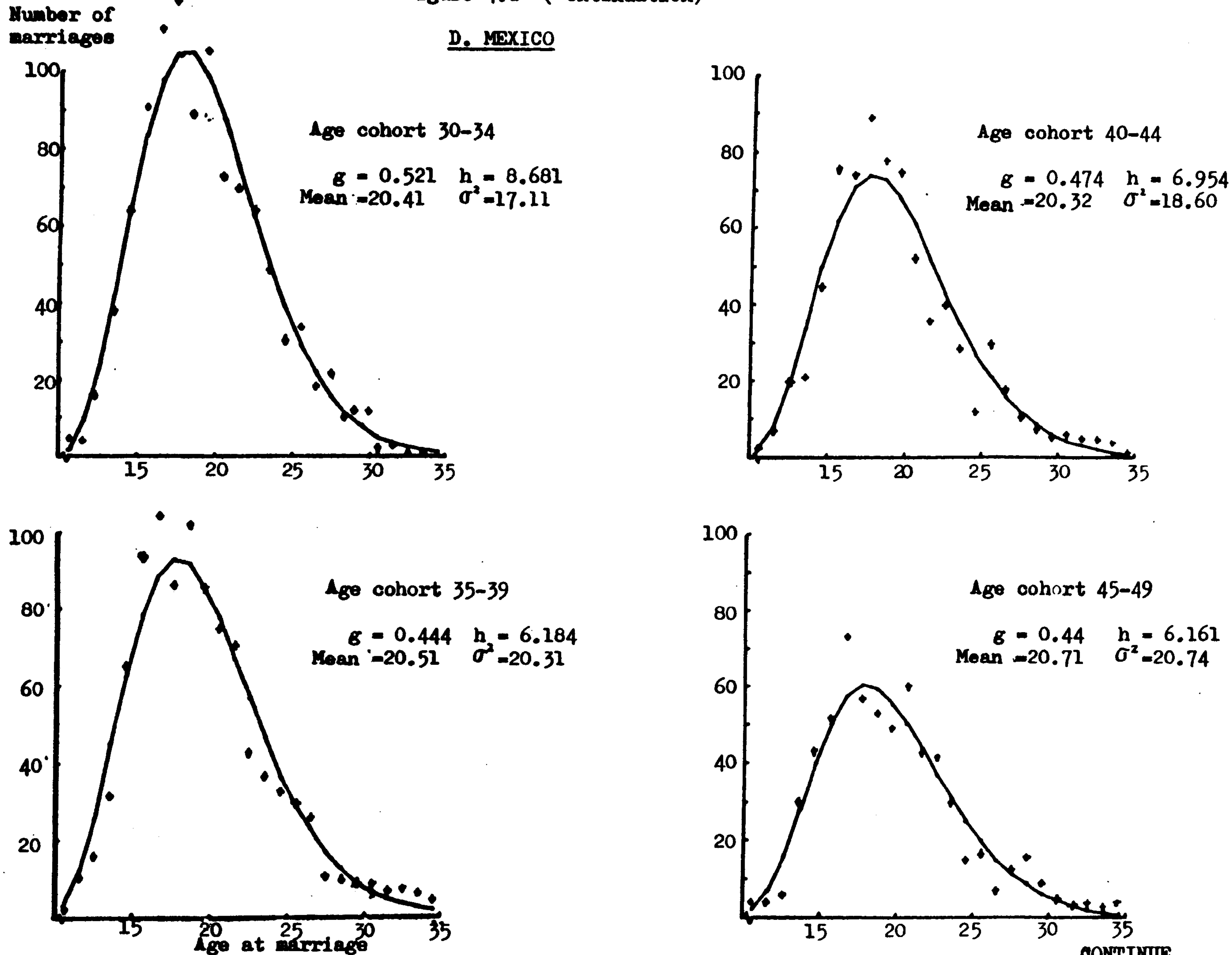
C. COSTA RICA



CONTINUE ...

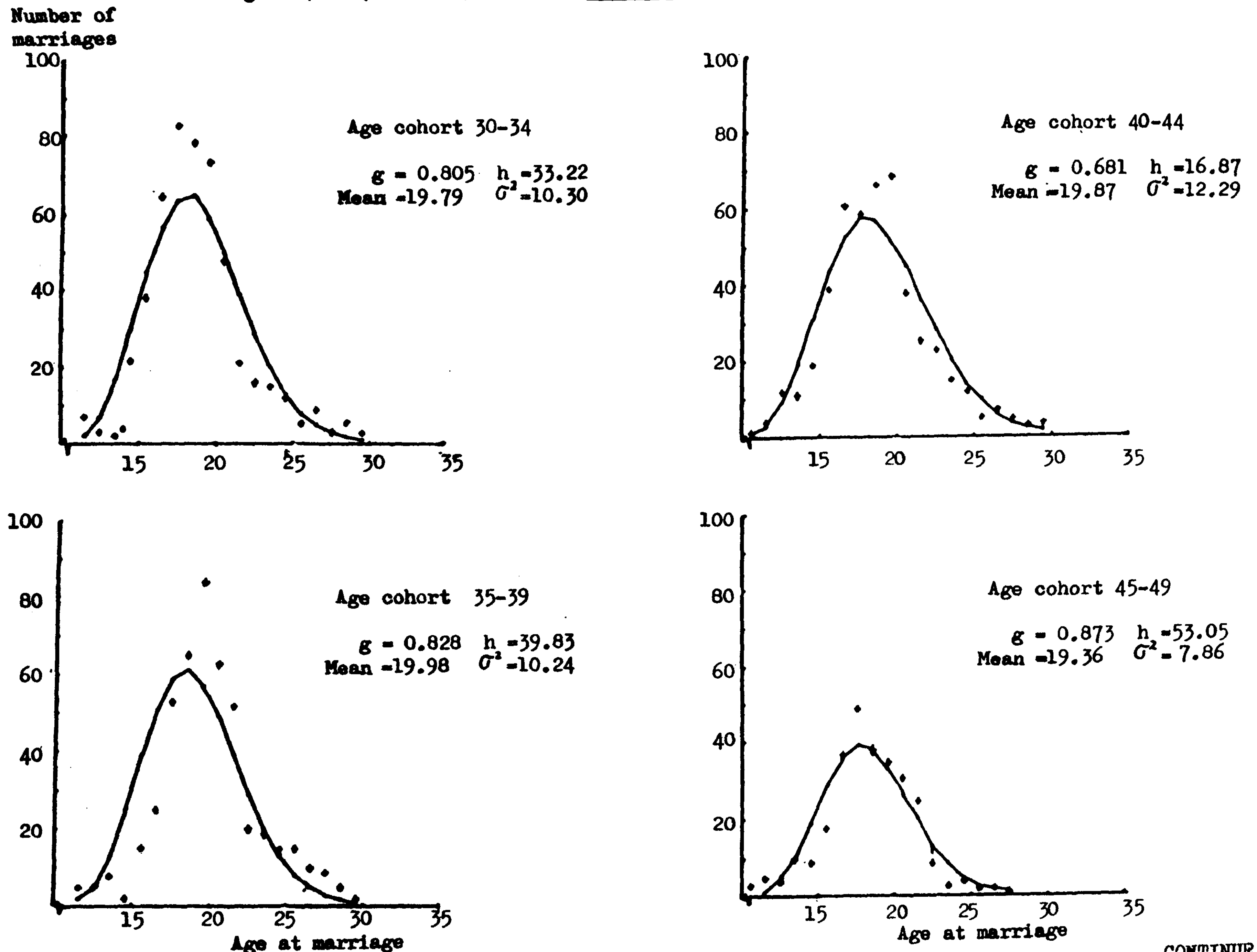
Figure 4.1 (Continuation)

D. MEXICO



CONTINUE ...

Figure 4.1 (continuation) **E. LESOTHO**



CONTINUE ...

Figure 4.1 (Conclusion)

F. REPUBLIC OF KOREA

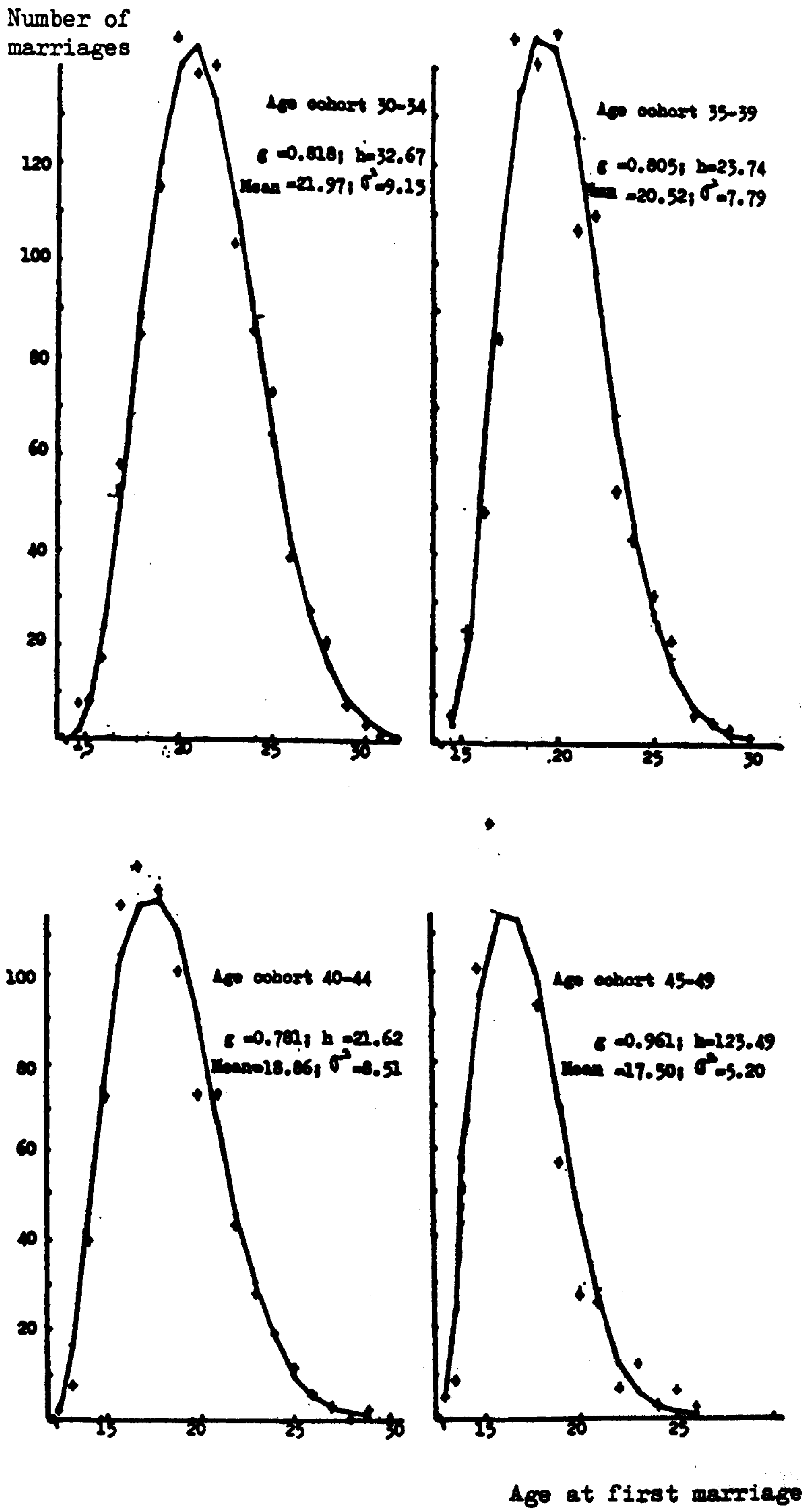
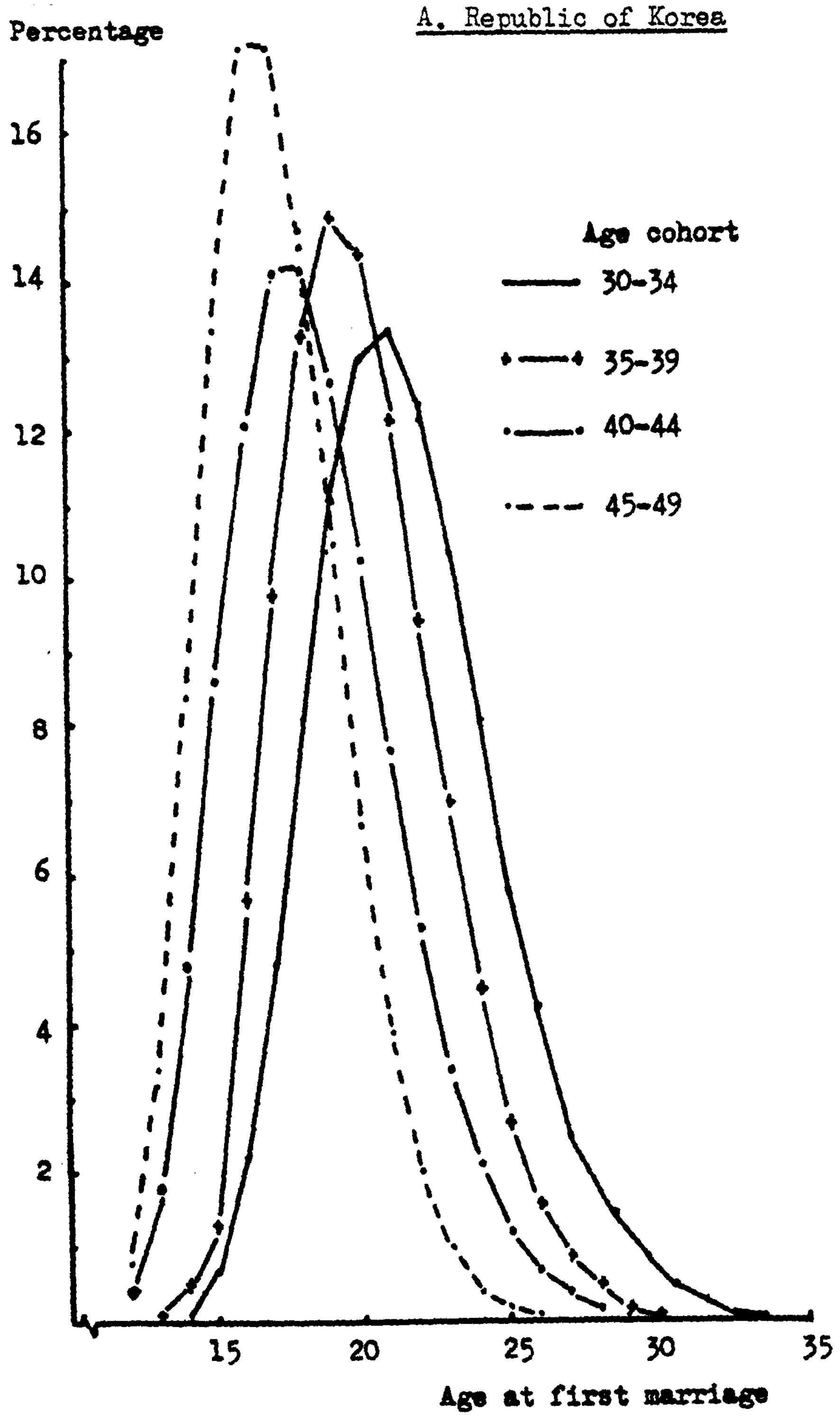


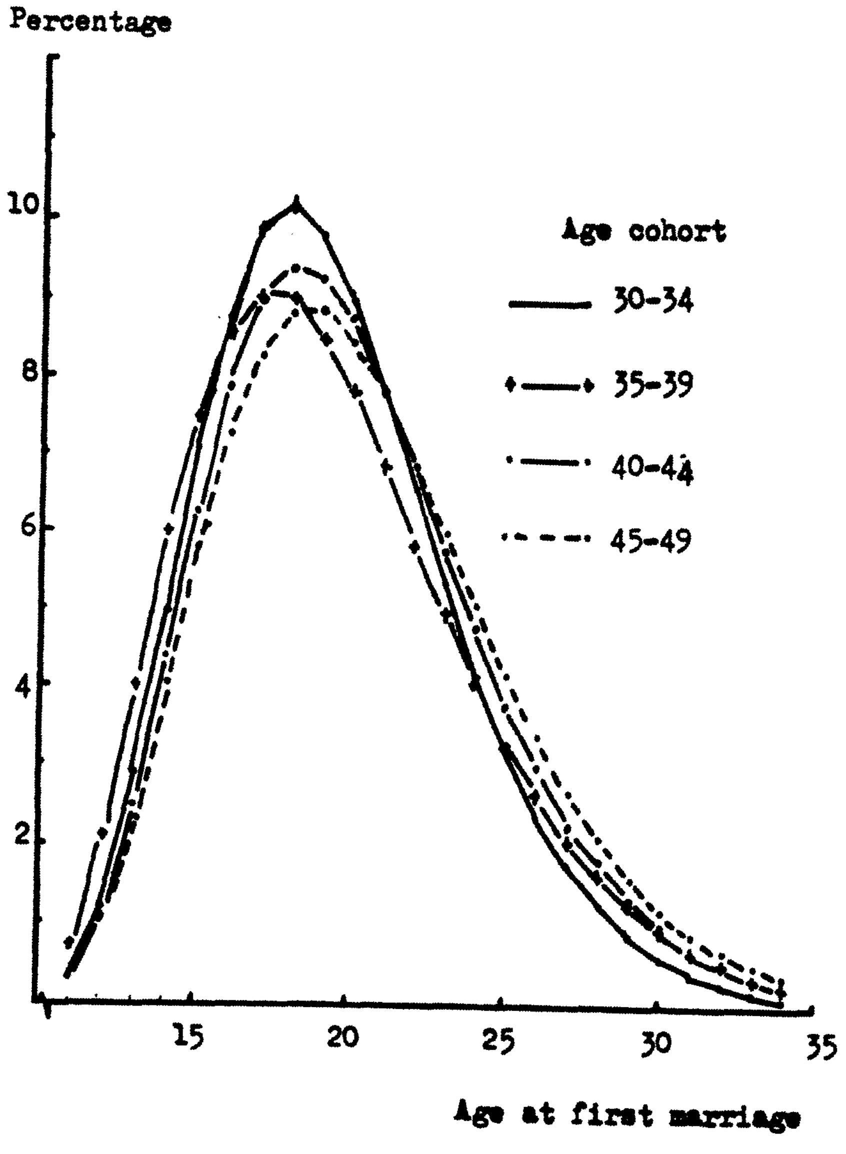
FIGURE 4.2: Per-cent Distribution of First marriages by Ages in Four Cohorts: Republic of Korea and Colombia



CONTINUE ..

FIGURE 4.2

B. Colombia



4.4 The model of fertility by age of the women and birth order

With appropriate models of nuptiality and of fertility by marriage duration and birth order, the derivation of a model of fertility by birth order and age of the women is straight forward.

Let assume that a particular age-cohort of women, designated by a, is followed. $M_i(a)$ is written for the probability that a woman of this cohort will marry in the i-th interval from entering the marriage market.

$D^*_i\{r/(r-1),a\}$ is the probability that a woman belonging to cohort <u>a</u> will have her r-th birth (having had r-1 in the preceeding intervals) in the i-th marriage duration interval (see equations 3.8 and 3.9).

Thus,

$$F_1(r,a) = M_1(a)\, D^*_1\{r/(r-1),a\} \qquad (4.7)$$

will be the probability of a r-th birth in the first interval (we assume that the marriage occurs at the beginning of the interval).

There are two ways in which a woman can achieve her r-th child in the second interval from entering the marriage market:

- i. by marrying in the first interval and then having her r-th child in the second interval from marriage; and

-ii. by marrying in the second interval from entering the marriage market and having her r-th child in the first interval from marriage; that is:

$$F_2(r,a) = M_1(a)\ D^*_2\{r/(r-1),a\} + M_2(a)\ D^*_1\{r/(r-1),a\} \qquad (4.8)$$

Accordingly,

$$F_3(r,a) = M_1(a)\ D^*_3\{r/(r-1),a\} + M_2(a)\ D^*_2\{r/(r-1),a\} + $$
$$+ M_3(a)\ D^*_1\{r/(r-1),a\} \qquad (4.9)$$

In general, the r-th birth in the N-th interval:

$$F_N(r,a) = M_1(a)\ D^*_N\{r/(r-1),a\} + M_2(a)\ D^*_{N-1}\{r/(r-1),a\} +$$
$$+ M_3(a)\ D^*_{N-2}\{r/(r-1),a\} + \dots +$$
$$\dots + M_{N-1}(a)\ D^*_2\{r/(r-1),a\} + M_N(a)\ D^*_1\{r/(r-1),a\} \qquad (4.10)$$

More compact:

$$F_N(r,a) = \sum_{i=1}^{N} M_i(a)\ D^*_{N-i+1}\{r/(r-1),a\} \qquad (4.11)$$

which defines the fertility model by birth order and age of the women as the convolution of the nuptiality function (given by the negative binomial) and the fertility model by duration of marriage (given by the beta binomial distribution).

On the basis of this model the average time exposure to the risk of dying for children by birth order and age of the mothers can be calculated. Such average exposures are the base for an indirect method of estimating child mortality from census (or survey) reports on the number of children ever born and children surviving to women, by age of the women and total children ever born, at the time of the interview. At the same time, by combining this fertility model with the model of mortality described in Chapter 2, correction factors to adjust the retrospective estimates of mortality for the effects of mother's age, birth order and birth spacing, can be obtained. Under certain circumstances such adjusting factors can facilitate the analyses of mortality trends. The next chapter describes the steps in the calculation process and the theoretical assumptions on which the procedure rests.

CHAPTER 5

Estimating Proportions of Children Surviving by Age and Parity of the Mother Using Models of Fertility and Mortality.

V. ESTIMATING PROPORTIONS OF CHILDREN DEAD BY AGE AND PARITY OF THE MOTHER USING MODELS OF FERTILITY AND MORTALITY

5.1 The calculation process.

In order to describe the calculation process it is convenient to disaggregate it, somehow arbitrarily, into successive stages. Such stages will be delineated briefly here, and a detailed explanation will be given in the following sections. The computer program written to execute the calculations is presented in Appendix 1. The necessary input data are:

1. The stopping rule ($S(r)$), expressed in term of the proportions of women willing and able to have r or more children.

2. The parameters a and b which characterize the fertility model by duration (Beta-binomial).

3. The parameters g and h which define the marriage distribution (negative-binomial), and a_0, which is the age at which women start to enter the marriage market.

The first step in the calculations is to obtain the average time exposure to the risk of dying for children of a given order, by current age (single years) of the mother. This is performed from line 53 to line 151 in the computer program, and explained in section 5.2.

The second step is the calculation of the age of the mother at birth. Given the children's average exposure to risk and the current age of the mother it should be possible, in principle, to obtain the mother's age at birth by subtraction. However, some adjustments are necessary in order to take into account the differences between women who, at the same age, have different numbers of children ever born. Such adjustments and the assumptions on which the calculations rest are explained in section 5.3. The execution of this step is performed from line 152 to line 261 in the computer program. The time exposures to risk are then estimated by subtracting the adjusted "ages at birth" from the "current ages" of the women, both measured from the same origin (performed from line 226 to line 273 in the program).

The last step consists in attaching the appropriate probabilities of surviving (according to pertinent life tables) to the average time exposures, in order to obtain proportions of children surviving. Average exposures by birth order, current age of the mother, and number of children ever born, have been obtained previously. With that information it is possible to calculate the proportion of children surviving classified by birth orders and total number of children ever born to their mothers, taking into account differential mortality by birth order, age of the mother at birth, and birth spacing, using the functional description of mortality introduced in Chapter 2. The computer program executes this step following instructions from line 283 to line 428. A more detailed description of this step of the calculation process is given in section 5.4.

5.2 Time-exposure to the risk of dying for children by birth order, age, and parity of the mothers.

The average time-exposure for children of a given birth order classified by mother's age and parity were obtained according to the following steps:

1) The fertility model by duration of marriage was calculated first. Interval length was taken as two years. The distributions of births by duration, for each order, were truncated at the 20th interval. The stopping rule was not included at this stage. An implicit assumption in the distribution of births obtained in this way is that all women would have attained their r-th birth after a sufficiently prolonged period, and will continue to have children indefinitely.

2) The distribution of births by order and duration of marriage (using an interval of two years) was transformed into a distribution by single years of marriage duration, by interpolating in the cumulated distribution using a third-degree polynomial function.

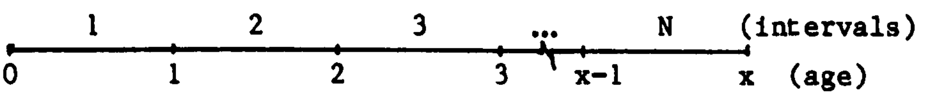
3) The calculation of the nuptiality model was done using a time interval unit equal to one year. Thus, time interval units for the distribution of marriage intervals coincide with those units for the marriage duration obtained in point 2.

4) The fertility model by age of the mother was then obtained by multiplying the model by duration by the nuptiality model, as described in Chapter 4, according to equation 4.11:

$$F_N(r,a) = \sum_{i=1}^{N} M_i(a)\, D^*_{N-i+1}\{r/(r-1),a\}$$

From now on, if no confusion is likely to arise from the notation, the index a indicating the particular birth cohort of women will be omitted for simplicity, writing only $F_N(r)$.

5) Having the distributions of birth by birth order and age interval at birth, it is possible to calculate average time-exposures to the risk of dying for children by birth order and age of the mothers. Taking age from an arbitrary origin at the onset of the nuptiality process:



The average time-exposure to the risk of dying for children of order r born to women aged x can be obtained as:

$$E_x(r) = \left[\sum_{N=1}^{x} (x-N+0.5)\, F_N(r)\right] \Big/ \left[\sum_{N=1}^{x} F_N(r)\right] \qquad (5.1)$$

assuming that, on average, children have been exposed for half a year during the interval in which the births occurred.

Similar calculations can be done for the distribution of births by duration of marriage and birth order, using the distribution obtained in Chapter 3, formula 3.9:

$$E^{T}(r) = \{ \sum_{n=1}^{T} (T-n+0.5)\ D_{n}^{*}[r/(r-1)] \} \ / \ \{ \sum_{n=1}^{T} D_{n}^{*}[r/(r-1)] \qquad (5.2)$$

where T is the marriage duration.

If the same fertility parameters (a and b in the beta-binomial) are used, the differences between $E_{x}(r)$ and $E^{T}(r)$ can be attributed, under certain assumptions, to the spread of ages at marriage introduced in $F_{N}(r)$ by the nuptiality function, as it is the only differing factor.

According to the assumptions on which these calculations were made (described in point 1), these exposures to the risk of dying correspond to children born to women who have reached <u>at least</u> parity r by that age (or marriage duration), since each of these births may have been followed by another one (or others). Therefore, the mean time-exposures obtained from equation 5.1 correspond to <u>all</u> children of a given order <u>r</u>, born to women aged x, who have had <u>at least</u> r children; they are not related uniquely to a fixed mother's parity. Some adjustments are necessary to adapt these estimates to resemble the type of cross-sectional data obtained from retrospective surveys.

Before proceeding further, it is convenient to specify some relations. Under the assumption that all women would eventually attain an r-th birth after a sufficiently prolonged duration of marriage, formula 4.11 can be used to estimate the number of women who, by age x, will have attained r or more births:

$$NB_x(r+) = \sum_{N=1}^{x} F_N(r); \qquad F_N(r) = 0 \quad \text{if } N<@ \; ; \tag{5.3}$$

obviously the probability of a woman having an r-th birth in age-interval N is zero for ages below a certain limit indicated by @ . In the same way, $NB_x[(r+1)+]$ gives the number of women who have had r+1 or more children by age x. Thus,

$$NB_x(r) = NB_x(r+) - NB_x[(r+1)+] \tag{5.4}$$

is the number of women with exactly r children at age x. Hence, in absence of a stopping rule, $NB_x(r)$ indicates the number of women in the birth cohort a who, at age x, have had r children and are waiting for the r+1 birth, which they will achieve after a certain time.

In a retrospective survey the reproductive experiences of different age cohorts of women are interrupted by the survey at their current ages, and the number of children achieved up to that age are recorded. For some women, with reported parity r at age x, the r-th child is only a stage since they subsequently will proceed to an (r+1)th child and eventually more. For other women parity r may be the final stage in the family building process either because at age x they might have became permanently sterile or because they have reached their desired family size and voluntarily stopped childbearing. In any case, the ages at which women have achieved (or may achieve) the r-th birth are spread over a certain range of ages. Part of such dispersion is caused by the spread of ages at marriage, and part is due to the different levels of fecundability among the women, and to chance factors. Women with higher parities at a given age will be those who have married earlier and/or progressed more quickly to bigger family sizes because of higher fecundability.

It is possible to calculate the average exposure to the risk of dying for r-th children born to women married over a range of ages (from formula 5.1) as well as for children born to women married all at the same age (formula 5.2). From these values, the "shifting back" to earlier ages at marriage for women who, by the same ages, have progressed to higher parity orders than the r-th one can be estimated indirectly. This is an important element in the estimation procedure to obtain the mother's age at birth of the r-th child, for women who have attained n children at the census date. This estimation procedure is developed in the next section.

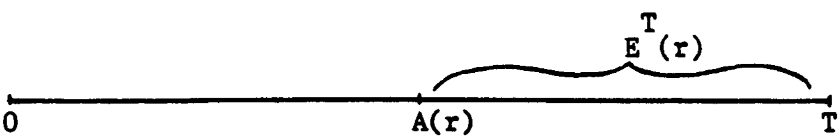
5.3 Age at birth of the r-th child for women who have borne n children.

The age of the mother at birth of the first, and then subsequent children, can be obtained through the following steps:

1) Supposing that all women marry at the <u>same</u> <u>age</u> (the fertility model by duration), the mean age at birth of the r-th child can be calculated as the age of the women at the survey minus the mean exposure to the risk of dying for children of r-th order. Let us write:

T for the age at the time of the survey (for practical purposes it will be measured from marriage), and

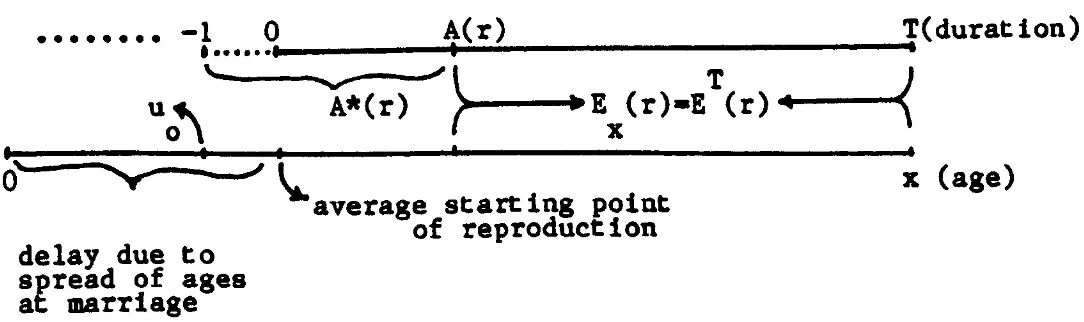
A(r) for the mean age at birth of the r-th child, for women aged T at the survey: $A(r) = T - E^{T}(r)$



Of course, under the assumptions of these models, A(r) is the mean age at birth of the r-th child, for women who have borne r or more. $E^{T}(r)$ represents an average exposure for r-th children, independent of whether or not they have been followed by another birth. Most likely, those children which have been followed by an (r+1)th, then by an (r+2)th, etc, were born earlier.

2) Let us consider now a situation in which the ages at marriage vary according to a nuptiality model (the fertility model by ages). Mean exposures to the risk of dying, $E_x(r)$, can be calculated from equation 5.1. There will be a certain duration T in the model by duration (with similar fertility parameters) for which $E^T(r)$ will be equal to $E_x(r)$. The difference x-T accounts for the spread of ages at marriage, the timing of nuptiality being the only differing factor in those calculations. This situation is illustrated in the following diagram (the meaning of u_o and $A^*(r)$, which appear in the diagram, will be explained in the following paragraphs).

Figure 5.1: Diagram describing the relation between equivalent time-exposures in the fertility model by ages and by marriage duration.



3) As it was pointed out before, an r-th child might have been followed by one or more children. If we take an arbitrary age, say x, for which the mean exposure for order r is $E_x(r)$, we can pick up in the model by duration the corresponding duration T, so that $E_x(r)=E^T(r)$ (in practice this will require interpolations between two appropriate durations). Under this condition (equal exposures for r-th children), the differences in the mean exposures for the higher orders between both models (for age x and duration T respectively), that is:

$$E_x(r+1) - E^T(r+1), \quad E_x(r+2) - E^T(r+2), \quad E_x(r+3) - E^T(r+3), \quad \ldots$$

show how far the starting point of reproduction shifts backwards to younger ages for those women who have progressed to higher parities, as a result of the spread of ages at marriage.

In general,

$$d^r(n,x) = E_x(n) - E^T(n) \tag{5.5}$$

under condition $E_x(r)=E^T(r)$; where $n = (r+1), (r+2), (r+3), \ldots$,

The values $d^r(n,x)$ estimate the additional time-exposure to the risk of dying for r-th children born to women who have reached family sizes of more than r children. These additional exposures come as a result of the extension-back in the starting point of reproduction for women with higher parities than r at age x.

The age of the women at birth of the r-th child can then be obtained under certain assumptions. Assuming that all women begin to have children at the same age, if age is measured from a convenient origin (u_o), which coincides with the average starting point of reproduction minus one year (see figure 5.1), then women aged $A^*(r)$ at birth of the r-th child would have been $A^*(r)/r$ at birth of the <u>first</u> child, on the assumption that the intervals between births are equal. The origin from which $A^*(r)$ is measured, $u_o = x-(T+1)$, makes the assumption of constant intervals between births closer to reality, as it allows for a period equivalent to the post-partum delay for first births.

Now the restriction of invariant starting point of reproduction for all women can be relaxed. Expresion 5.5 can be used for estimating the variations in that starting point, according to the family size (total number of children) attained by the women at a given age. Let denote the mean age of the mothers at birth of their r-th children, for women currently aged x who have attained at least n children, by $MA^r\{n+,x\}$. This value can be estimated as:

$$MA^r\{n+,x\} = r\ [A^*(n)/n] - d^r(n,x) \qquad (5.6)$$

4) Now it is necessary to obtain the mean age of the mothers at birth of their r-th children for women with <u>exactly n children</u> at a given time, say at the census date. The mean ages for women with n or more children and also for those with n+1 or more can be obtained from equation 5.6. The proportions of women who have attained n or more

($NB_x(n+)$) and n+1 or more children ($NB_x[(n+1)+]$) at age x, are given by expression 5.3. Equation 5.4 provides the proportion of women who have exactly n children ($NB_x(n)$). From these values, the mean age at birth of the r-th child for women with only n children at age x, $MA^r(n*,x)$, can be obtained as a weighted average:

$$MA^r(n*,x)=\{MA^r(n+,x)\ NB_x(n+) - MA^r[(n+1)+,x]\ NB_x[(n+1)+]\}/NB_x(n) \quad (5.7)$$

5) As the stopping rule was not included in the calculation process leading to formulas 5.6 and 5.7, the values obtained from equations 5.6 and 5.7 only apply to women <u>who will continue to have children</u>. They take no account of women who cease to have children because of sterility, broken marriage or deliberate decision. Also according to the assumptions of these models, the stage at which women stop their family building process is independent of the age, as the stopping rule, S(r), depends only on the number of children attained. In a survey, the women reported as having n children at current age x are a mixture of those who are waiting for the next child and those who, at that order, have reached their final family size and stopped childbearing altogether. The mean age at birth of the r-th child for women who have had n or more, $MA^r\{n+,x\}$, provides and estimate for the age at birth of the r-th child for women who, at the census, have already reached their final family size, say n. In the surveyed population, women who stopped at n were in a position to progress to higher orders (proportion $NB_x(n+)$ in the model) on the basis of their fertility timing but, whatever the reasons, stayed at n. Those

couples who were willing and able to have more than n children either have already moved to higher parity orders (proportion $NB_x[(n+1)+]$ in the model) or, at woman's age x, are still waiting for the birth of their next child (proportion $NB_x(n)$ in the model). For those women still waiting for the birth of the (n+1)th child, $MA^r(n*,x)$ would be a reasonable estimate for the mean age at birth of the r-th child. Hence, a weighted average of the values $MA^r(n*,x)$ and $MA^r(n+,x)$, obtained from the model, can be taken as an estimate for the mean ages at birth in the surveyed population. The appropriate weighting factors are given by the stopping rule S(n) which describes the proportions of women willing and able to have n or more children. The ratio S(n+1)/S(n) indicates the proportion of women who, having achieved an n-th birth, will eventualy have another one, and 1- S(n+1)/S(n) is the proportion of those women staying at n. Therefore, the pertinent weighted average would be:

$$MA^r(n,x)=[S(n+1)/S(n)]\ MA^r(n*,\ x) + \{1-[S(n+1)/S(n)]\}\ MA^r(n+,x) \qquad (5.8)$$

Then, $MA^r(n,x)$, is the estimate for the mean age at birth of the r-th child for women who have born n children by current age x. In the model, age x is measured from the point at which women begin to enter the marriage market, hence, conventional ages from birth can be obtained by fixing that origin. On the other hand, the ages at birth are also measured from an arbitrary origin, which is the adjusted average starting point of reproduction (with allowance for an equivalent to post-partum delay for first births). Thus, the time-exposure to the risk for each order by parity and age of the mother is the span of time from that arbitrary origin to the "present" moment (corresponding to current ages of the women), minus the age of the women at the birth of their children (which is measured from the same origin). This can be seen more clearly by referring to the diagram presented in figure 5.1. The $MA^r(n,x)$ value obtained from equation 5.8 corresponds, in figure 5.1, to A*(n) after been adjusted for the variations in the total number of children achieved by the women.

The age at which women begin to enter the marriage market (a_0) is represented in figure 5.1 by the arbitrary origin zero. For a given population, where such age is a_0, the age scale can be transformed to refer to ages from birth, by just adding a_0. The adjusted starting point of reproduction, represented by u_o, is calculated as x-(T+1). Therefore, the current age, x, as well as the the mean age at birth, $MA^r(n,x)$, can be expressed in terms of ages from birth as:

- current age = $x + a_0$
- age at birth = $a_0 + u_o + MA^r(n,x)$

and, the time-exposure to risk = (current age) - (age at birth)

From these values the proportions of children surviving to women by age and number of children ever born can be obtained as described in the next section.

5.4 Proportions of children surviving by current age and parity of the mothers.

The information on number of children ever born and number of children still alive, collected in so many censuses and surveys around the world can be tabulated by age and parity order of the women. Proportions of children surviving, or its complement, can then be obtained by age and parity of the mothers.

The models used in this research can facilitate the analysis of mortality by age and parity of the mothers from those proportions. Under the assumption of constant fertility and mortality, the number of children of a given order born t years ago to women currently aged x, and the proportions surviving after n years from birth, are the same as those for children born t-m years ago to women currently aged x-m ($n<t-m$), the only adjustment needed being that of the growth rate effect, in order to take account of the changes in population size. Proportions of children dead can be obtained from the mean exposures to the risk by birth order, age, and parity of the women, calculated in the previous section, by combining the mean exposures with appropriate life tables. The following paragraphs explain the steps required for these calculations.

1) The proportion of children surviving from birth up to exact age t is given by the life table function $l(t)$, with radix equal to one. Let us write $t_x^{i,n}$ for the mean time-exposure to the risk of dying for i-th children born to women aged x who have borne n children. From the previous section, this value is obtained by subtracting age at birth from current age:

$$t_x^{i,n} = x - MA^{i}(n,x) \tag{5.9}$$

The proportion of children surviving, according to a life table with the characteristics described in Chapter 2 will be $l(t_x^{i,n})$.

2) Since each woman with parity n would have borne a child for each birth order up to n (multiple births are treated as single births), the average (over all orders) proportion of children surviving to those women at age x is:

$$P(n,x) = \{ \sum_{i=1}^{n} l(t_x^{i,n})\} / n \tag{5.10}$$

3) Proportions of children surviving by five-years-age groups and parity of the mothers can be calculated as a weighted average:

$${}_5P_x(n) = \{ \sum_{j=0}^{4} e^{-0.02\ j}\ P(n,x+j)\} / \{ \sum_{j=0}^{4} e^{-0.02\ j} \} \tag{5.11}$$

where 0.02 is the rate of population growth, which was kept constant at two per cent per year in all the calculations, and x is the lower limit of the five year age interval.

4) If we write ${}_5D_x(n)$ for the proportion of children who have died among the children ever born to women with parity n in the age group x,x+4, then:

$$ {}_5D_x(n) = 1 - {}_5P_x(n) \qquad (5.12) $$

5) In order to obtain the average proportion (over all parity orders) of children surviving to women at a given age x, it is necessary to take into account the proportions of women reaching parity order n by single years of age, as the proportions P(n,x) have to be weighted by the number of children borne to each woman.

Let $NB_x^*(n)$ denote the proportion of women who have borne n children at age x, under the stopping rule S(r). Equation 5.3 gives the proportion of women having n or more children at age x ($NB_x(n+)$), assuming that all women would achieve an n-th child after a suficiently prolonged marriage duration, and will continue to have children. Then, taking into account the stopping rule:

$$ NB_x^*(r) = \{ S(n) \cdot NB_x(n+)\} - \{ S(n+1) \cdot NB_x[(n+1)+]\} \qquad (5.13) $$

and the average proportion (over all orders) of children surviving to women aged x:

$$ P_x = \{ \sum_{n=1}^{\gamma} n \cdot NB_x^*(n) \cdot P(n,x)\} / \{ \sum_{n=1}^{\gamma} n \cdot NB_x^*(n)\} \qquad (5.14) $$

The highest number of children ever born to women aged x is indicated by γ in equation 5.14.

6) The average proportion of children surviving by five year age groups of the women is then calculated by averaging the proportions P(x) in a similar way as was done in relation 5.11:

$$_5P_x = \{ \sum_{j=0}^{4} e^{-0.02\,j}\, P(x+j)\} \,/\, \{ \sum_{j=0}^{4} e^{-0.02\,j} \} \qquad (5.15)$$

The analysis of the proportions obtained from equation 5.15 is the subject of Chapter 6. Under the assumption that the level of child mortality is invariant by the mother's age, "expected" proportions of children surviving are calculated. The expected proportions are then compared with the "model" proportions, which consider differential mortality by mother's age, birth order, and birth spacing. In this way the differential mortality which affects children born to younger mothers is evaluated, so the retrospective estimates can be adjusted.

The proportions obtained from equation 5.11 are studied in Chapter 7. Particular attention is given to the variation in the average time exposure by parity within each age group of the mothers. The conclusions drawn from these analyses indicate that retrospective information on the number of children ever born and children surviving by age group of the mothers can be safely used for studying the differentials in child mortality by family size. The study of such differentials is illustrated with two applications using census data from Bolivia and Guatemala.

CHAPTER 6

The Impact of Differential Mortality by Mother's Age and Birth Order on the Retrospective Estimates from Indirect Methods.

VI. THE IMPACT OF DIFFERENTIAL MORTALITY BY MOTHER'S AGE AND BIRTH ORDER ON THE RETROSPECTIVE ESTIMATES FROM INDIRECT METHODS

6.1 Introduction

Throughout this chapter the term "simulated" proportion is used to denote those results in which the mortality risk is a function not only of the child's age but also depends on the birth order and mother's age, as determined by the functional description of mortality, defined in Chapter 2 (equations 2.1, 2.2, 2.3). "Standard" proportion indicates results where the mortality function varies with age of the child only, following the Brass' General Standard pattern. For a given age group and parity, both measures (simulated and standard) refer to the same time-exposure, therefore their logits can be related through the linear equation in the logit life table system. "Expected" proportions of surviving children can be calculated under the assumption that the overall mortality level is the same as that implied in the simulated proportions, but the risks are invariant with birth order and mother's age, depending only on the child's age (following the standard pattern). In this way the difference between the simulated and the expected proportions would indicates the effects of the differential mortality associated with the reproductive patterns.

In relation to these reproductive patterns, three main factors have been explicitly included in the calculations, hence their effects can be controlled and analysed independently: the stopping rule, the patterns of nuptiality, and the pace of marital fertility.

The stopping rule determines the absolute level of fertility and the patterns of family formation. A minute analysis of the effects of such patterns on infant and child mortality is not within the aims of this study. For our purposes the variations in the proportions of children surviving, resulting from changes in the stopping rule, have to be interpreted as the quantitative effects on the proportions surviving, associated with the fertility structure by family size. In other words, those changes describe how the simulated proportions of children surviving vary when the number of births by order changes for a given pattern of mortality, nuptiality, and marital fertility pace.

The nuptiality pattern plays an important role. Very early nuptiality implies that a significant number of births may occur at young ages, where the risks are high. In societies where little or no family planning is practised this effectively means that large family sizes may be attained at relative young ages, a situation which heightens the risks considerably.

In the context of these analyses the effects of the pace of marital fertility can be observed by fixing the stopping rule and the nuptiality pattern, while changing the marital fertility distribution. To illustrate how changes in the fertility pace may affect the distribution of births we can point out that, for a slow fertility pace (parameter p around 0.5 for interval units of two years), it is expected that about 80-85 per cent of the first births would occur within four years from marriage, around 20-25 per cent of third births within six years, and about 11-15 per cent of sixth births during the first twelve years. For a fast pace (p at about 0.75), around 90 per-cent or slightly more of the first births would occur within the first four years of marriage, 45-50 per cent of third births within six years, and about 25-30 per cent of sixth births during the first twelve years of marriage. Faster fertility pace means that a higher proportion of high order births is reached at a given age. Thus more births will be happening in high concentration categories, affected by higher mortality. It is convenient to remember that under the assumptions of these models either litle or no birth control occurs, or birth control operates by stopping after a given family size has been attained, but not through birth spacing.

6.2 Differences in the levels of mortality from retrospective estimates associated with the age group of the respondents.

In order to analyse the variations in the level of mortality associated with the age group of respondents, it is necessary to adopt a base with which the different estimates can be compared. Such base must represent a fair mixture of the mother's ages at birth and birth orders that occurred in the population. The proportion of children surviving to women aged 40-44 was taken as the base for these comparisons. This group was preferred, rather than the age group 45-49, because the assumptions on which the models are based become less realistic as the extremes of the reproductive interval are appoached, therefore near those boundaries the results are less reliable. On the other hand, the relatively few births to women older than 45 which are ruled out, as group 40-44 is adopted, are unlikely to modify the "overall" level of mortality significantly. Since the scale factor K, in equations 2.1, 2.2, and 2.3, was given the value one in all simulations, the overall level of mortality in the simulated proportions should be close to that from the standard. However, as the distribution of births differs from the one used for specifying the functions A(y), P(r), and C(c) (equations 2.4, 2.5, 2.6, in section 2.5, Chapter 2), changes in the number of births occurring in the different subclasses would introduce some variations in the overall mortality level. The next paragraph explains how these variations are accounted for in the calculation procedure.

From the simulated and standard proportions of children surviving to women aged 40-44 the, alpha value in the one parameter logit life table system is calculated:

$$\alpha = \text{logit}\ (P_6^{sm}) - \text{logit}\ (P_6^{sd}) \qquad (6.1)$$

where $\text{logit}(P) = 0.5 \ln\{(1-P)/P\}$,

P_6^{sm} is the simulated proportion, and

P_6^{sd} is the standard proportion for age group 40-44, this alpha represents the overall level of mortality in the simulated population.

With the parameter alpha and the standard proportions for each age group (P_i^{sd}), "expected" proportions can be obtained (P_i^*):

$$P_i^* = 1/\{1+ \exp 2[\alpha+\mathrm{logit}(P_i^{sd})]\} \qquad (6.2)$$

where $i = 1,2, \ldots$ indicates age groups 15-19, 20-24, ...

These "expected" values represent the proportions of children that would survive to mothers by groups of ages, if mortality is constat by age of the mother, birth order, and concentration, and the overall mortality level is equal to that from the simulated proportions.

Finally, ratios from the expected to the simulated proportions of children dead are calculated:

$$C_i = (1-P_i^*)/(1-P_i^{sm}) \qquad (6.3)$$

Three patterns of marital fertility, corresponding to p equal to 0.857, 0.643 and 0.429, ($p=a/(a+b)$, equation 3.12) were combined with three patterns of nuptiality and four stopping rules, to produce a number of simulated proportions of children surviving from which the values C_i, presented in table 6.1, were obtained.

The nuptiality patterns were defined by the following parameters:

negative binomial		marriage distribution	
g	h	mean age	variance
0.54	6.0	17.4	11.00
0.48	6.5	19.6	16.93
0.46	7.7	22.7	22.20

g and h are the parameters of the negative binomial distribution, and the mean and variance were obtained from equations 4.3, and 4.4, with age at onset of nuptiality (a_0) equal to 11 and 12, and marriages assumed to happen at the mid point of the marriage duration interval.

The four patterns of fertility by birth order, corresponding to total fertility rates at about 7.0, 6.0, 5.0, and 4.0 respectively, are defined by the following stopping rules, S(r):

TFR	r												
	1	2	3	4	5	6	7	8	9	10	11	12	13
7.00	.94	.90	.87	.82	.74	.66	.56	.47	.38	.28	.18	.11	.05
6.00	.92	.88	.82	.75	.68	.59	.48	.35	.22	.13	.07	.05	.03
5.00	.90	.86	.79	.70	.56	.43	.30	.21	.12	.06	.03	.02	.01
4.00	.89	.79	.66	.53	.42	.28	.18	.13	.06	.03	.015	.008	.004

These patterns were derived from observed distributions of women by completed family sizes.

The main patterns of variation in C_i, as each one of these three factors change, can be observed in table 6.1.

Table 6.1: Factors C_i by age group for different fertility and nuptiality patterns.

Fertility		C_i				
Level (TFR)	Pace (p)	15-19	20-14	25-29	30-34	35-39
I. Nuptiality distribution: $\bar{x} = 17.4$ $\sigma^2 = 11.0$						
7.00	0.857	0.883	0.953	0.983	0.970	0.971
	0.643	0.871	0.961	1.003	1.020	1.007
	0.429	0.852	0.960	1.023	1.048	1.026
6.00	0.857	0.826	0.898	0.933	0.939	0.973
	0.643	0.821	0.912	0.958	0.986	0.990
	0.429	0.815	0.922	0.987	1.018	1.014
5.00	0.857	0.765	0.838	0.884	0.925	0.971
	0.643	0.768	0.855	0.914	0.960	0.980
	0.429	0.769	0.875	0.947	0.994	1.004

(Continue)

Table 6.1 (continuation)

Fertility		C_i				
Level (TFR)	Pace (p)	15-19	20-14	25-29	30-34	35-39
	Nuptiality distribution:	$\bar{x} = 19.6$		$\sigma^2 = 16.9$		
6.00	0.857	0.802	0.923	1.048	1.044	1.001
	0.643	0.801	0.925	0.998	1.030	1.012
	0.429	0.795	0.933	1.018	1.052	1.030
5.00	0.857	0.745	0.858	0.981	0.997	0.985
	0.643	0.754	0.875	0.954	0.999	1.003
	0.429	0.753	0.886	0.976	1.020	1.019
4.00	0.857	0.713	0.833	0.954	0.981	0.985
	0.643	0.725	0.851	0.956	0.989	1.001
	0.429	0.727	0.863	0.957	1.009	1.016
	Nuptiality distribution:	$\bar{x} = 22.7$		$\sigma^2 = 22.2$		
6.00	0.857	0.793	0.951	1.138	1.172	1.070
	0.643	0.785	0.946	1.075	1.111	1.050
	0.429	0.774	0.950	1.058	1.089	1.058
5.00	0.857	0.751	0.903	1.074	1.122	1.055
	0.643	0.746	0.902	1.026	1.071	1.038
	0.429	0.739	0.909	1.016	1.059	1.043
4.00	0.857	0.725	0.878	1.040	1.094	1.043
	0.643	0.722	0.881	0.998	1.055	1.033
	0.429	0.718	0.890	0.994	1.048	1.037

The features which clearly stand out in table 6.1 are, in first place, that children born to women under 20 suffer heavier than overall mortality. Secondly, the proportion of children dead to women 20-24 reflects also a level of mortality higher than that for all children. Children born to women in this group when they were younger (under 20), probably have a significant impact on this average, even when numerically they are a minority. For older groups the situation varies according to the nuptiality and fertility characteristics, but the C_i coefficients are generally close to one.

With respect to variations with nuptiality and fertility, the response to changes in such patterns are not simple. It is clear that the most important changes take place when moving from one nuptiality pattern to another. The level of fertility determined by the proportions having r or more children, according to the stopping rule, also produce significant changes in the C_i ratios. However, the variations in C_i due to changes in different factors are not uniform by age groups.

For a given level of fertility and a nuptiality pattern, as the pace of fertility became slower, the relative excess of mortality affecting children born to women under 20 increases, while the change in C_i for age groups over 20 generally moves in the opposite direction, sometimes with very little change. Since in the early reproductive ages the situation can vary very little (independently of the average fertility pace all births will be affected by the adverse impact of mother's age while there would be little time for moving on to higher orders in spite of a faster pace), such variation seems more likely to reflect the effect of the fertility pace on the overall mortality, with which the group 15-19 is compared, rather than changes within the group 15-19 itself. The simulated proportions of children dead reflect a level of mortality between 10 and 25 per cent higher than the level expected under the assumption of constant mortality by mother's age and birth order. When a higher proportion of women progress to high parities (stopping rule for TFR=7), the adjusting factor to make mortality in this group comparable to that for all births is closer to one: the advantage of lower risks associated with ages older than 20 is somehow counteracted in part by more births in higher concentration groups and higher orders.

The results in table 6.1 show C_i values consistently lower than one for the age group 20-24. For a given nuptiality pattern C_i becomes lower (bigger correction) when the level of fertility is lower. Similar to the case of age group 15-19, it seems likely that this is more the result of variations in the overall level rather than in group 20-24 itself. A faster pace in marital fertility increases the relative mortality level for this group. However, such variation is only moderate, reaching a maximum of about three per cent.

For ages above 25 the C_i values fluctuate around one, and in most cases denote only a small correction. The only case in which the adjusting factor for age group 25-29 indicates a correction of the order of ten per cent is in that of very early nuptiality, very fast pace and a TFR equal to five.

6.3 An example using data from Peru

The same data used in figure 1.1 to illustrate the analysis of trends in childhood mortality from indirect estimates will be used here. Table 6.2 presents the proportions of children dead, the coefficients C_i and the estimated q(x) and alpha (α) values adjusted and unadjusted. The derivation of q(x) from the proportions of children dead is explained in several papers quoted already in Chapter 1. Only the adjustment of the retrospective estimates to account for differential mortality by age group of respondents is considered here.

The C_i values for 1972 were selected from the panel in table 6.1 with mean age at marriage at 19.6, a TFR of 6.00, and a fast pace (p=0.857). For 1976 and 1977 the C_i correspond to the same nuptiality and pace parameters as for 1972, but an average of the values for TFR=6.00 and TFR=5.00 was taken, following the decline that occurred in fertility.

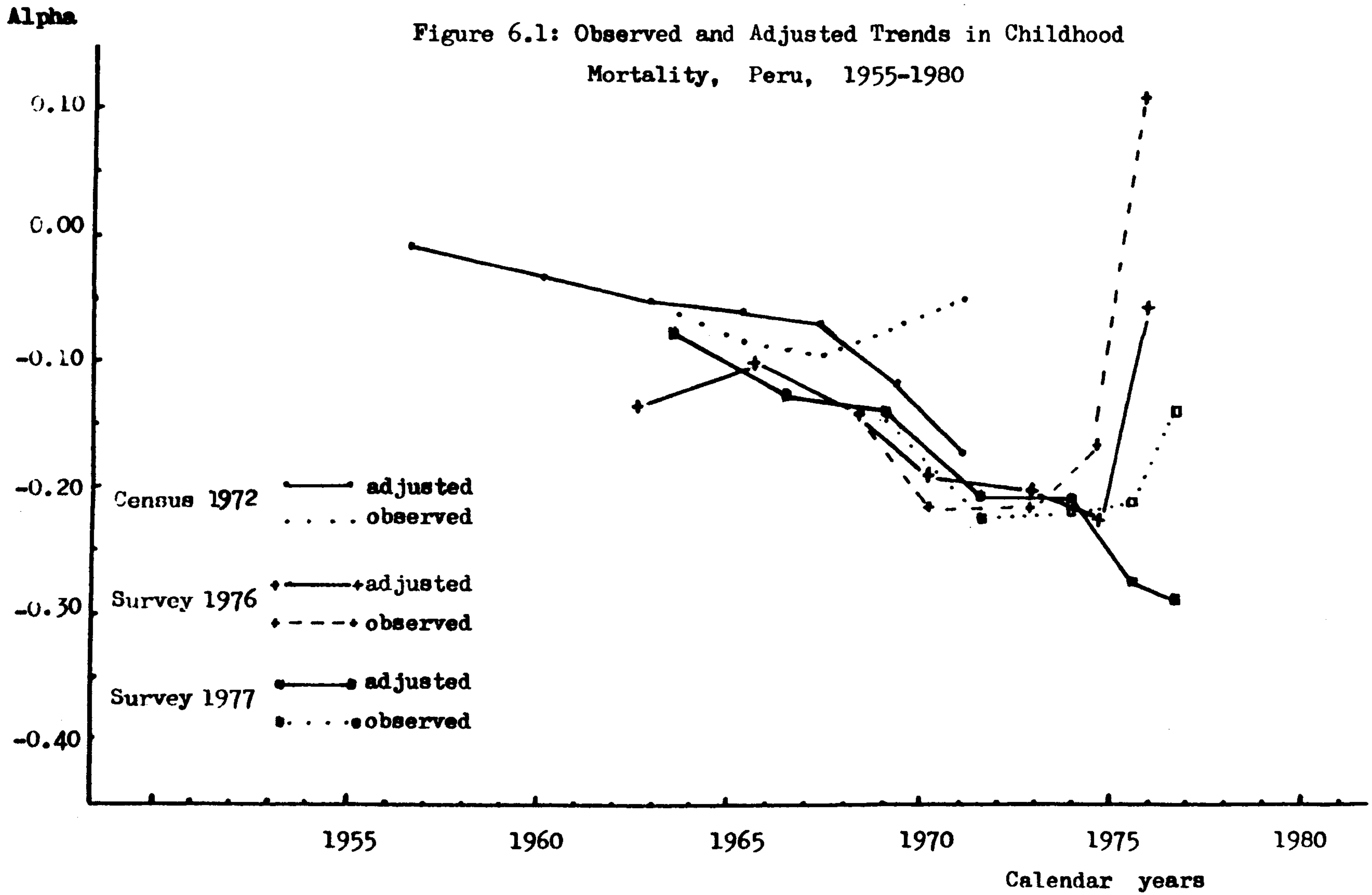
Figure 6.1 shows the adjusted and unadjusted trends. Except for the group 15-19 from the 1976 survey, the adjusted values fit very well into the overall trend indicated by all the points from the three data sources together. The upward turn which appears in the unadjusted estimates from age groups 15-19 and 20-24 are the result of the higher risks experienced by the children born to these women. The analysis of the data from the 1976 survey by sex (Instituto National de Estadística -INE-, 1978) reveals that the very high mortality for children born to women under 20 reflect an anomalously high proportion of female children reported dead. Such a sex differential is not consistent with the sex differential observed in the reports from other age goups and other data sources. The cause of this anomaly is not clear. However, if the overall sex differential is maintained and the level of mortality from reports on male children is accepted, the estimate would be consistent with all the rest of the points.

An attempt to adjust estimates from the younger age groups may not be as successful in other cases as it was in this example. Women who marry and have children very early represent a highly selective group in some societies. If that selectivity is associated with mortality, then the children born to these women will be affected by a different level of mortality, not only because of the reproductive pattern, but also because of other factors which determine a level of mortality not comparable with that for the whole population.

Table 6.2: Indirect Estimates of Childhood Mortality Levels (α) from Proportions of Children Dead, Adjusted for Differential Mortality by Age Group of Respondents, PERU.

Age of respondent	Proportion dead	C_i	Unadjusted q(x)	Unadjusted α	Adjusted q(x)	Adjusted α
			1972 Census			
15-19	0.1475	0.802	0.1377	-0.050	0.1104	-0.176
20-24	0.1755	0.923	0.1731	-0.067	0.1597	-0.115
25-29	0.1873	1.048	0.1835	-0.091	0.1923	-0.062
30-34	0.2042	1.044	0.2021	-0.850	0.2110	-0.058
35-39	0.2312	1.001	0.2307	-0.052	0.2309	-0.052
40-44	0.2562		0.2493	-0.038	0.2492	-0.038
45-49	0.2905		0.2821	-0.012	0.2820	-0.012
P_2/P_3 = 0.530						
			1976 Survey			
15-19	0.1517	0.774	0.1777	0.101	0.1374	-0.052
20-24	0.1333	0.891	0.1463	-0.167	0.1303	-0.234
25-29	0.1435	1.015	0.1493	-0.215	0.1515	-0.206
30-24	0.1589	1.021	0.1646	-0.211	0.1680	-0.198
35-39	0.1922	0.993	0.2008	-0.141	0.1994	-0.145
40-44	0.2215		0.2273	-0.099	0.2273	-0.099
45-49	0.2240		0.2301	-0.149	0.2301	-0.149
P_2/P_3 = 0.395						
			1977 Survey			
15-19	0.1090	0.774	0.1256	-0.103	0.0971	-0.248
20-24	0.1246	0.891	0.1359	-0.210	0.1211	-0.276
25-29	0.1431	1.015	0.1484	-0.218	0.1505	-0.211
30-34	0.1563	1.021	0.1616	-0.222	0.1649	-0.210
35-39	0.1932	0.993	0.2014	-0.139	0.2000	-0.143
40-44	0.2141		0.2192	-0.122	0.2192	-0.122
45-49	0.2510		0.2571	-0.075	0.2571	-0.075
P_2/P_3 = 0.407						

Figure 6.1: Observed and Adjusted Trends in Childhood Mortality, Peru, 1955-1980



6.4 Conclusions

It seems, at this stage, that table 6.1 contains enough information for most cases in which an adjustment of the proportions of children dead would be needed. When information on children ever born and surviving is available, usually some knowledge about the pattern of nuptiality and the level of fertility (enough to locate the situation about some panel in table 6.1) is also available. A precise knowledge of nuptiality and fertility is not necessary. Information on the pace of fertility may be more scanty in some cases, but the results are not very sensitive in relation to this parameter. In any case, in absence of any information, using the medium pace (p=0.643) would be reasonable, considering that the margin of error which this may produce is in most cases within two per cent. This margin seems quite acceptable taking into account the approximations and the simplifying assumptions inherent in the calculation of C_i.

Considering that these results are only approximate, in most cases an attempt to adjust retrospective estimates for age groups above 30 (or even 25-29 in some cases) would not be justified. Children born to these women are already a fair mixture of orders and ages at birth. Several other factors may produce differences as important as the differentials by reproductive patterns associated with the selection which, at later stages of the reproductive period, still may remain. The biases in age groups 15-19 and 20-24 are in most cases very important and the correction would be of an order of magnitude far bigger than the margin of error which may arise from simplifying assumptions and from imperfect approximations in the parameters used for selecting the multipliers C_i.

The assertion made above, that estimates from age groups 15-19 and 20-24 are biased, implicitly assumes that these values are used for estimating the level of mortality affecting all children in the population, which indeed is the purpose of such statistics in most cases. However, strictly speaking, these are in themselves estimates which measure the mortality of children born to women under 20 and 25 years of age respectively, and for some particular purposes it may be of interest to know the level of mortality for these specific groups. Obviously, in such cases the estimates have to be used at face value, any adjustments (except to transform the proportions of children dead into conventional life table functions) are pointless and incorrect.

Finally it should be mentioned that if women having children at very young ages are a selected group, such selection may be associated with an altogether different level of child mortality and the correction proposed here would not solve the problem of comparability with the mortality level for the whole population.

CHAPTER 7

Analysis of the Proportions of Children Surviving by Age of the Mother and Parity.

VII. ANALYSIS OF THE PROPORTIONS OF CHILDREN SURVIVING BY AGE OF THE MOTHER AND PARITY.

7.1 Introduction

As explained in Chapter 5, proportions of children surviving have been calculated by birth order, by family size (parity), and mother's single years of age. The analysis carried out in Chapter 6 required aggregation of birth orders and family sizes, thus the results depended on the stopping rule which weighted the family sizes on the averaging.

In this chapter the proportions of children surviving are analysed by family size and age of the mother. In relation to those of the previous chapter, this type of analysis has the advantage of being independent from the stopping rule, as each family size is taken separately. In first place attention will be given to the variations in the average time-exposures by family size. The mean exposures obtained from model distributions are compared with exposures obtained from observed birth distributions. Then the simulated and standard proportions of surviving children will be compared, and the practical implications of the findings will be discussed.

7.2 Mean time-exposure to risk by family size and mother's age.

Table 7.1 presents the average time exposure to risk by age and parity, and the simulated and standard proportions of children surviving, for three different situations of nuptiality and fertility. Results analogous to these, but for a wider range of nuptiality and fertility patterns, are shown in Appendix 2.

The effects of birth concentration, birth order and age of the mother are ostensible. An idea of the magnitude of such effects is provided by the difference between the simulated and the standard proportions in each age-parity group. The simulated proportions decrease dramatically at very high parities, and are lower than the standard ones at ages under 20 for any family size. A more detailed discussion of these variations is carried out in the next section.

A remarkable feature is the stability of the mean time exposure by parity for any given age group, according to the results from these models. At first sight this stability looks rather surprising. One may expect that bigger families have been attained by starting childbearing earlier and this, in turn, would be associated with longer average exposures at higher parities. Information from birth-histories can be used to obtain analogous statistics, allowing us to compare these results with those from real data.

Table 7.1: Mean time-exposure to risk, standard, and simulated proportions of children surviving by age of the mother and family size, for three different patterns of nuptiality and fertility.

Family size	Age Group						
	15-19	20-24	25-29	34-34	35-39	40-44	45-49
A. Fertility p = 0.5; nuptiality: g=0.56, h=5.5, $\bar{x}$=16.6, σ^2 =9.1							
Mean time-exposures							
1	1.84	3.51	5.73	8.50	11.73	15.24	19.13
2	1.90	3.39	5.37	7.88	10.88	14.61	17.92
3	2.02	3.25	5.08	7.67	10.91	14.82	17.99
4	2.47	3.16	4.89	7.42	10.87	14.95	18.08
5	0.0	3.46	4.75	7.32	10.96	15.20	18.33
6	0.0	3.72	4.64	7.25	11.07	15.43	18.56
7	0.0	0.0	4.76	7.16	11.22	15.69	18.82
8	0.0	0.0	5.08	6.99	11.29	15.88	19.02
9	0.0	0.0	0.0	6.76	11.03	15.76	18.98
10	0.0	0.0	0.0	7.23	10.71	15.65	18.94
Simulated proportions of children surviving							
1	0.772	0.786	0.793	0.793	0.790	0.781	0.763
2	0.727	0.756	0.777	0.798	0.794	0.783	0.767
3	0.688	0.724	0.761	0.780	0.789	0.780	0.763
4	0.661	0.685	0.735	0.766	0.770	0.771	0.757
5	0.0	0.658	0.702	0.736	0.751	0.748	0.741
6	0.0	0.628	0.659	0.708	0.726	0.731	0.721
7	0.0	0.0	0.629	0.671	0.696	0.706	0.702
8	0.0	0.0	0.601	0.629	0.665	0.679	0.680
9	0.0	0.0	0.0	0.597	0.629	0.653	0.660
10	0.0	0.0	0.0	0.571	0.597	0.632	0.643
Standard proportions of children surviving							
1	0.814	0.782	0.766	0.755	0.746	0.735	0.718
2	0.811	0.783	0.767	0.757	0.748	0.738	0.724
3	0.807	0.785	0.769	0.758	0.748	0.737	0.724
4	0.798	0.786	0.770	0.759	0.748	0.736	0.724
5	0.0	0.782	0.771	0.759	0.748	0.736	0.722
6	0.0	0.779	0.772	0.759	0.748	0.735	0.721
7	0.0	0.0	0.771	0.760	0.747	0.734	0.720
8	0.0	0.0	0.769	0.760	0.747	0.733	0.719
9	0.0	0.0	0.0	0.761	0.748	0.734	0.719
10	0.0	0.0	0.0	0.759	0.748	0.734	0.719

(continue)

Table 7.1 (Continuation)

Family size	Age Group 15-19	20-24	25-29	34-34	35-39	40-44	45-49
B. Fertility p = 0.5 ; nuptiality: g=0.6, h=8.0, $\bar{x}$=18.5, σ^2 =10.							
Mean time-exposures							
1	1.49	2.82	4.75	7.31	10.31	13.78	17.51
2	1.65	2.80	4.46	6.82	9.37	12.95	16.52
3	1.89	2.78	4.32	6.54	9.34	13.14	16.68
4	0.00	2.91	4.19	6.38	9.31	13.46	16.94
5	0.00	3.34	4.05	6.23	9.20	13.53	17.09
6	0.00	0.00	4.22	6.11	9.22	13.71	17.30
7	0.00	0.00	4.54	5.87	9.02	13.55	17.22
8	0.00	0.00	0.00	5.84	9.02	13.73	17.41
9	0.00	0.00	0.00	6.29	8.81	13.65	17.39
10	0.00	0.00	0.00	6.81	8.77	13.72	17.90
Simulated proportions of children surving							
1	0.790	0.801	0.804	0.802	0.796	0.787	0.771
2	0.748	0.774	0.793	0.806	0.800	0.790	0.773
3	0.711	0.740	0.778	0.792	0.799	0.787	0.769
4	0.0	0.715	0.753	0.779	0.782	0.781	0.763
5	0.0	0.687	0.718	0.754	0.764	0.759	0.753
6	0.0	0.0	0.687	0.727	0.742	0.742	0.731
7	0.0	0.0	0.654	0.691	0.720	0.723	0.714
8	0.0	0.0	0.0	0.652	0.685	0.700	0.697
9	0.0	0.0	0.0	0.626	0.652	0.678	0.678
10	0.0	0.0	0.0	0.597	0.620	0.652	0.659
Standard proportions of children surviving							
1	0.829	0.791	0.771	0.759	0.749	0.740	0.726
2	0.822	0.791	0.773	0.761	0.752	0.743	0.731
3	0.812	0.792	0.774	0.762	0.752	0.742	0.730
4	0.0	0.789	0.775	0.763	0.752	0.741	0.729
5	0.0	0.784	0.776	0.763	0.753	0.741	0.728
6	0.0	0.0	0.775	0.764	0.753	0.740	0.727
7	0.0	0.0	0.772	0.765	0.753	0.741	0.728
8	0.0	0.0	0.0	0.765	0.753	0.740	0.727
9	0.0	0.0	0.0	0.763	0.754	0.741	0.727
10	0.0	0.0	0.0	0.761	0.754	0.740	0.726

(continue)

Table 7.1 (Continuation)

Family size	Age Group 15-19	20-24	25-29	34-34	35-39	40-44	45-49

C. Fertility p=0.857 ; nuptiality: g=0.4, h=4.0, $\bar{x}$=22.0, σ^2=18.8

Mean time-exposures

Family size	15-19	20-24	25-29	34-34	35-39	40-44	45-49
1	0.98	1.65	2.39	4.15	8.66	13.79	18.77
2	1.39	2.02	2.87	3.87	6.22	10.72	15.92
3	0.00	2.32	3.24	4.32	5.98	8.97	14.15
4	0.00	2.87	3.53	4.82	6.57	8.94	13.13
5	0.00	0.00	0.00	5.16	7.13	9.57	13.28
6	0.00	0.00	0.00	6.05	7.57	10.36	14.07
7	0.00	0.00	0.00	0.00	7.87	11.13	14.81
8	0.00	0.00	0.00	0.00	8.46	11.28	15.01
9	0.00	0.00	0.00	0.00	9.59	12.22	16.08
10	0.00	0.00	0.00	0.00	0.00	12.77	16.26

Simulated proportions of children surviving

Family size	15-19	20-24	25-29	34-34	35-39	40-44	45-49
1	0.820	0.837	0.842	0.824	0.802	0.787	0.765
2	0.776	0.797	0.823	0.831	0.812	0.790	0.774
3	0.0	0.761	0.801	0.824	0.817	0.797	0.779
4	0.0	0.750	0.770	0.801	0.806	0.796	0.780
5	0.0	0.0	0.0	0.771	0.784	0.782	0.769
6	0.0	0.0	0.0	0.749	0.762	0.763	0.754
7	0.0	0.0	0.0	0.0	0.734	0.742	0.735
8	0.0	0.0	0.0	0.0	0.711	0.720	0.718
9	0.0	0.0	0.0	0.0	0.682	0.690	0.695
10	0.0	0.0	0.0	0.0	0.0	0.672	0.676

Standard proportions of children surviving

Family size	15-19	20-24	25-29	34-34	35-39	40-44	45-49
1	0.851	0.822	0.799	0.775	0.754	0.740	0.720
2	0.833	0.807	0.790	0.778	0.763	0.748	0.733
3	0.0	0.801	0.785	0.774	0.764	0.753	0.739
4	0.0	0.790	0.782	0.770	0.762	0.753	0.742
5	0.0	0.0	0.0	0.768	0.760	0.751	0.742
6	0.0	0.0	0.0	0.764	0.758	0.749	0.739
7	0.0	0.0	0.0	0.0	0.757	0.747	0.736
8	0.0	0.0	0.0	0.0	0.755	0.747	0.736
9	0.0	0.0	0.0	0.0	0.751	0.745	0.732
10	0.0	0.0	0.0	0.0	0.0	0.743	0.732

Mean time-exposures were calculated from birth histories collected in three fertility surveys conducted within the WFS programme. They are presented in table 7.2 (number of cases and standard deviation for each cell are presented in Appendix 3). Panel D of this table shows the average exposures obtained from models which resemble patterns of nuptiality and fertility by order and marriage duration prevailing in Latin American countries. These models were selected on the basis of the results obtained in Chapter 3 and Chapter 4. The stability in the mean exposures by family size within each age group is also remarkable in these three countries. The the broad patterns of variation in the time-exposures observed in these three countries are followed closely by the exposures obtained from the models. Although this is not proof that the results obtained from the models are free of errors or biases, it does show that they are very plausible and provide a reasonable basis for analysing the variations in the time exposures by age and parity.

In a closer analysis, comparing the model values with the observed ones, it is apparent that some systematic differences appear in the younger age groups. The observed exposures are shorter than the expected (according to the model), for the smaller family sizes. After a given family size (2 children, sometimes 3), the observed exposures change very little, and that happens in the model as well. This difference can be explained, at least partially, in terms of nuptiality changes. There is evidence that cohorts under the age of 30 at the time of the surveys experienced a delay in ages at first marriage, in

Table 7.2 Average Exposure to Risk by Mother's Age and Parity Calculated from the National Fertility Surveys (WFS) from Mexico, Peru and Colombia, and from Models.

Parity order	Age Group						
	15-19	20-24	25-29	34-34	35-39	40-44	45-49
				A. Mexican Fertility Survey			
1	1.05	1.77	2.97	6.07	8.59	14.51	20.45
2	1.72	2.32	3.73	6.28	9.99	15.08	19.11
3	2.16	3.10	4.67	6.84	9.71	15.98	17.80
4	2.64	3.78	5.30	7.69	11.05	15.84	19.16
5		4.52	5.72	7.58	10.62	14.73	19.77
6		4.46	5.96	7.98	10.37	14.08	19.64
7		5.41	6.52	8.22	10.91	14.35	18.52
8			6.90	8.45	10.88	14.57	18.06
9			6.86	8.81	11.36	14.30	17.43
10				8.59	11.20	13.77	18.01
				B. Peru National Fertility Survey			
1	1.02	1.63	2.85	7.29	9.49	15.70	15.99
2	1.74	2.45	3.81	6.35	10.50	15.91	20.58
3	2.43	3.22	4.82	6.57	10.08	14.45	19.77
4	2.92	3.99	5.21	7.14	10.21	14.22	18.87
5		4.30	5.44	7.19	10.59	14.05	18.22
6		4.78	6.12	8.11	10.54	14.31	18.26
7		6.15	6.59	8.12	10.36	14.47	18.23
8		5.31	6.45	8.30	10.53	14.01	17.38
9			7.40	8.74	10.63	13.47	18.58
10				10.38	11.01	13.96	18.27

(continue)

Table 7.2 (Continuation)

Parity order	Age Group						
	15-19	20-24	25-29	34-34	35-39	40-44	45-49
				C. Colombian Fertility Survey			
1	1.05	1.88	3.56	6.34	9.00	15.20	20.49
2	1.52	2.66	4.42	6.90	10.00	13.73	20.65
3	2.43	3.31	5.31	7.37	10.86	15.92	19.71
4	2.43	4.03	5.69	8.19	11.47	14.05	18.71
5		4.28	5.93	8.53	12.21	13.88	18.57
6		4.50	6.24	8.21	11.56	15.53	18.93
7			6.34	8.43	11.48	15.37	16.60
8				9.31	10.96	13.94	17.18
9				8.84	12.18	14.22	17.63
10					10.76	13.54	18.29
		D. Model Distribution:		$p=0.786$,	$\bar{x}=19$,	$\sigma^2=15$	
1	1.55	2.59	4.39	8.49	13.64	18.63	23.51
2	1.79	2.83	4.20	6.63	10.75	16.10	20.86
3	2.07	2.95	4.46	6.62	9.56	14.33	19.17
4		3.13	4.49	6.61	9.38	13.52	17.71
5		3.64	4.66	7.15	10.37	14.56	18.24
6			4.73	7.05	10.42	14.68	18.21
7			5.09	6.57	9.76	14.11	17.60
8				7.26	10.93	15.49	18.90
9				7.53	10.39	15.11	18.54
10					10.02	14.90	18.40

comparison with older cohorts. In the model presented in table 7.2 the nuptiality pattern corresponds to the experience of those older cohorts, whose distributions were fitted in Chapter 4. Younger women have been marrying at later ages, and indeed the observed pattern of exposures, increasing with family size at young ages, is compatible with the patterns obtained from models with later and more spread nuptiality and fast fertility pace. Obviously, in a situation of changing nuptiality a unique set of models would not be able to describe appropriately the average exposures for all age groups, and a pattern of later nuptiality than the one used in this model is more appropriate for cohorts 15-19, 20-24 and perhaps 25-29. However, it is likely that this inconsistency is not entirely the cause of nuptiality changes. Such pattern again appear in data from Lesotho, where the evidence about nuptiality changes is not so convincing, as we will see later.

Another aspect in which the observed exposures in table 7.2 differ from the results obtained from the model concerns the average exposure for one child families at older ages. Particularly in the age groups 40-44 and 45-49, the observed exposures are in some cases significantly different from the model ones. There is a tendency in the model to give longer exposures for children born to women who at older ages have attained only one or two children. This is particularly marked in regimes which combine very early and concentrated nuptiality with fast fertility pace (as can be observed in Appendix 2, where results from a series of models are presented). It appears also where there is early and concentrated nuptiality and moderate fertility pace, and in intermediate nuptiality and very fast fertility pace. This pattern does not appear so clear in the data from the three countries presented in table 7.1, and is not very strong either in the model presented in panel D of that table.

Within the logic imposed by the model description, in the context of populations with early and concentrated nuptiality and fast fertility pace, even women who marry very late (within such context) would have been married already by their early twenties, and had their first child within a few years from marriage, at most. Therefore, when the women had reached their forties, first children must have been exposed to the risk of dying for twenty years or more. Frequently in this type of population women who have only one child are a selected group and do not adjust to the general patterns which characterize the population as a whole. If this group marry substantially later than the rest (that is, their behaviour is not properly described by the nuptiality model), then the results from the models would exagerate the time-exposure to risk for children born to these women. That may be the case in Latin American countries.

In the case of Lesotho the picture in relation to older ages is different. Table 7.3 presents the mean time-exposures for Lesotho, as calculated from birth histories (WFS data). The pattern of longer exposures for one (and to a lesser extent two) child families for older women is very marked (number of cases per cell and standard

Table 7.3: Mean time-exposures to risk calculated from data from the Lesotho Fertility Survey (WFS), and from models.

Parity order	Age Group 15-19	20-24	25-29	34-34	35-39	40-44	45-49
			A. Lesotho Fertility Survey				
1	1.11	1.81	4.19	8.03	15.89	19.43	26.83
2	1.90	2.57	4.28	7.78	12.77	17.04	21.42
3		3.32	4.47	7.42	11.46	16.05	21.39
4		4.60	5.11	6.77	9.50	13.34	19.26
5		4.86	6.02	7.41	9.83	14.01	18.04
6			7.45	7.76	10.06	13.53	17.49
7			5.35	8.52	9.94	12.80	17.21
8				9.39	10.29	12.91	16.98
9				10.39	10.78	13.06	17.67
10					10.00	14.69	16.63
	B. Model: $p=0.642$, $g=0.58$, $h=5.5$, $\bar{x}=17.0$, $\sigma^2=8.1$						
1	1.81	3.38	5.90	9.96	15.11	20.08	25.05
2	1.91	3.38	5.38	8.13	11.80	16.15	20.05
3	2.05	3.31	5.22	7.91	11.40	15.49	18.75
4		3.26	5.16	7.91	11.70	15.76	18.86
5		3.54	4.98	7.85	11.91	16.05	19.07
6		3.79	4.82	7.76	12.00	16.22	19.24
7			4.89	7.57	11.98	16.31	19.36
8			5.19	7.18	11.61	16.12	19.21
9				7.04	11.77	16.38	19.51
10				7.49	11.37	16.24	19.43

deviations are showed in Appendix 3). A similar picture appears in the time exposures presented in panel B, which have been simulated by models representing a pattern of early and concentrated nuptiality and moderate fertility pace.

The pattern of shorter exposures for smaller family sizes at young ages, observed in the three countries in table 7.2, appears in Lesotho as well. Although younger women did report later ages at first marriage in the Lesotho Fertility Survey, Timaeus and Balasubramanian (1984) dismissed the possibility of changes in the age at first marriage, explaining the difference in terms of misdating of first marriages: older women apparently declared earlier dates at first marriages than the actual ones. Numerically the differences between observed and model exposures may not be very big in some cases, but they are relevant because of the high rate of change in the mortality function at these young ages. The assumption that births occur at the mid-point of the year-interval introduces a small bias, as they would be concentrated towards the end of the interval in the first stages of the fertility distribution, but that would not explain all the difference.

The tendency to give longer exposures than those generally observed, for first children at young ages of the mother, may indicate some lack of flexibility in the methodology to cope with the fast changes which take place at early stages of childbearing. Adolescent subfecundity, which is not incorporated into the models, would produce patterns of differences similar to those which appear between the observed and the model results. This point will be discussed again in the next section.

In practical terms neither the differences at the beginning of the reproductive period nor the cases of one child families at ages above 40 represent a very serious problem. Such cases comprise a small proportion of the children born in societies where these techniques may be applied. The group of women having only one child at the end of their reproductive lives would be highly selective in many respects, and both the level of mortality and reproductive patterns would be most likely associated with other factors, which would set them quite apart from the average population. On the other side, the fast rate of change of the birth distribution at a very early stage of the reproductive period is very dificult to describe with a simple model. Therefore, with the necessarily simplified methodology that had to be used in this type of analysis, it is unlikely that attempts to improve the model representation in this respect would have met with any reasonable success.

7.3 Practical implications of these findings

The break-down of the proportions of children surviving by age of the mother and number of children ever born, obtained from census or survey data, frequently shows a substantial decrease in the proportion of children surviving as the total family size increases. Attempts to interpret these variations have been hampered by the fact that they could be connected either with higher risks for higher orders and birth concentrations or with longer exposures associated with higher parities, or a combination of both. The results analysed in the previous section indicate that differences in time-exposure play a small part in those variations. This is particularly true for age groups above 25, where the average exposures are fairly stable, and at the same time the rate of change with age of the child in the mortality function is low. For these age groups of the mothers it is quite safe to interpret the variation in the proportions of children surviving, from one family size to another, as the result of differential mortality, assuming constant time exposures. As the figures in table 7.1 show, the proportions surviving are almost constant by family size for a given age group when mortality is a function of the child's age only (standard proportions).

As for the younger age groups, on the assumption that the data from the four countries observed here is accurate, a more precise description of the observed patterns of variation in the time-exposures by family size (in tables 7.2 and 7.3), would require a more spread and later nuptiality distribution than that for the older age groups. The differences between the observed and the model time-exposures in these age groups may be connected to changes in nuptiality, but that pattern may also respond to the effects of adolescent subfecundity, which are not incorporated into the fertility model.

The model representation can be adjusted to take account of the factors mentioned above by using age-parity specific indices to relate the simulated time-exposures from the models to the observed data. A more spread and perhaps a little later nuptiality pattern would be able to resemble the variations on the fertility distribution by ages (therefore on the exposures to risk) caused by adolescent subfecundity. This adjustment, and that required for a situation where nuptiality changes from one cohort to another, would be implicit in the calculation procedure if the age-parity specific time-exposures, estimated from models, are fitted to the observed data by using age-parity specific fertility indices. The time exposure by mother's age and parity depends on the shape of the birth distribution by order and age. The true birth distribution is not known, but observed age-parity specific indices can be used as indicators for the shape of that distribution in the same way as P_1/P_2 and P_2/P_3 have been used in the original method. However, at this stage it seems that such efforts would not be justified. On the one hand it is unreasonable to expect that the models would describe the real situation with regards to the average exposures to the risk with a precision of one tenth of a year or two. On the other hand the data itself would probably be affected by a bigger margin of error than that.

In any case, the inspection of the simulated proportions of children surviving, presented in table 7.1, leads us to the conclusion that the effects of differential mortality are far bigger than the differences which may arise from variations in the exposures, even in the case of age groups 15-19 or 20-24, where the rate of change in mortality with age of the child is higher, and the relative error in the time exposures more important. Notwithstanding, limiting the analysis only to children born to women aged 25 or more is not very restrictive. Such analyses would cover a substantial proportion of the children ever born to the surveyed women, since the number of children born to women under 20, or even under 25, do not represent an important proportion of the total children a woman would have in countries of high fertility, and the number of children in one child families for women over 40 is very small. The proportions of children dead by age of the mother from Bolivia, 1976 Census, and from Guatemala, 1970 Census, are analysed in the next section.

7.4 Estimating differential mortality by family size from retrospective information on number of children ever born and children surviving.

In the light of the discussions in the previous section, it seems that the most sensible way to use this information is first to estimate the overall level of mortality in the traditional way, from information referring to all children, and then to use ratios between parity-specific proportions of children dead to estimate relative risks by family size.

Part A of table 7.4 presents the results of such analysis using data from Bolivia, 1976 Census. Part B shows the results from Guatemala, 1970 census. The probabilities of dying before reaching exact ages x were derived from the proportions of children dead by using Brass's multipliers. These values were then expressed in terms of the alpha parameter (α) in the one-parameter-logit system, to make them comparable. The time location was also calculated (T). These results are showed in the first panel, of part A, and of part B, for the respective contries. The proportions of children dead by family size are presented in the second panel.

Relative risks by family size were calculated taking the risks for all children as the base. The relative risks by family size, that is, the ratios from the proportions of children who have died, by family size, to that proportion for all children for the same age group of mothers, are presented in the third panel for the respective country, table 7.4.

Table 7.4: **Indirect estimates of child mortality and relative risks by family size. Bolivia, 1976 and Guatemala, 1970.**

	Age Group						
	15-19	20-24	25-29	34-34	35-39	40-44	45-49
				A. Bolivia, 1976 Census			
Total							
Di	0.1587	0.2024	0.2301	0.2532	0.2772	0.3036	0.3315
q(x)	0.1604	0.2080	0.2309	0.2556	0.2824	0.3025	0.3298
α	0.039	0.047	0.054	0.067	0.084	0.095	0.101
T	1.17	2.63	4.51	6.71	9.10	11.79	15.06
Family size				Proportions of children dead			
1	0.0946	0.0765	0.0778	0.0787	0.0730	0.0815	0.1129
2	0.1925	0.1431	0.1222	0.1066	0.1258	0.1566	0.1517
3	0.3056	0.2153	0.1648	0.1490	0.1546	0.1926	0.2121
4	0.3889	0.2977	0.2322	0.1869	0.1802	0.1874	0.2277
5	0.3714	0.3230	0.2793	0.2354	0.2364	0.2235	0.2482
6		0.3507	0.3286	0.2820	0.2499	0.2710	0.2967
7		0.3617	0.3254	0.2879	0.2736	0.2840	0.2920
8			0.3896	0.3422	0.3191	0.3047	0.3255
9			0.4514	0.3592	0.3355	0.3384	0.3441
10				0.3906	0.3684	0.3523	0.3588
				Relative risk			
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	0.60	0.38	0.34	0.31	0.26	0.27	0.34
2	1.21	0.71	0.53	0.42	0.45	0.52	0.46
3	1.93	1.06	0.72	0.59	0.56	0.63	0.64
4	2.45	1.47	1.01	0.74	0.65	0.62	0.69
5	2.34	1.60	1.21	0.93	0.85	0.74	0.75
6		1.73	1.43	1.11	0.90	0.89	0.90
7		1.79	1.41	1.14	0.99	0.94	0.88
8			1.69	1.35	1.15	1.00	0.98
9			1.96	1.42	1.21	1.11	1.04
10				1.54	1.33	1.16	1.08

Table 7.4 (continuation)

	Age Group						
	15-19	20-24	25-29	34-34	35-39	40-44	45-49
	B. Guatemala, 1970 Census						
Total							
Di	0.1016	0.1389	0.1683	0.1843	0.2123	0.2382	0.2608
q(x)	0.0947	0.1369	0.1648	0.1823	0.2117	0.2317	0.2531
α	-0.262	-0.206	-0.156	-0.149	-0.108	-0.086	-0.086
T	1.67	3.03	4.99	7.25	9.69	12.49	15.91
Family size	Proportions of children dead						
1	0.0518	0.0419	0.0400	0.0364	0.0642	0.0612	0.0646
2	0.1310	0.0954	0.0801	0.0746	0.0826	0.1015	0.1274
3	0.2011	0.1409	0.1082	0.0895	0.1009	0.1171	0.1242
4	0.2391	0.1955	0.1583	0.1281	0.1240	0.1401	0.1648
5	0.3600	0.2468	0.1865	0.1465	0.1578	0.1564	0.1812
6		0.3319	0.2261	0.2003	0.1768	0.1828	0.2031
7		0.2457	0.2682	0.2126	0.1928	0.2040	0.2467
8			0.3275	0.2562	0.2331	0.2543	0.2449
9			0.4505	0.2793	0.2494	0.2468	0.2667
10			0.3895	0.2865	0.2968	0.2872	0.2884
	Relative risk						
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	0.51	0.30	0.24	0.20	0.30	0.26	0.25
2	1.29	0.69	0.48	0.40	0.39	0.43	0.49
3	1.98	1.01	0.64	0.49	0.48	0.49	0.48
4	2.35	1.41	0.94	0.70	0.58	0.59	0.63
5	3.54	1.78	1.11	0.79	0.74	0.66	0.69
6		2.39	1.34	1.09	0.83	0.77	0.78
7		1.77	1.59	1.15	0.91	0.86	0.95
8			1.95	1.39	1.10	1.07	0.94
9			2.68	1.52	1.17	1.04	1.02
10			2.31	1.55	1.40	1.21	1.11

The differentials are dramatic. The risks for small families are in some cases a fourth or a fifth of the overall risk for all children, while on the other hand the biggest family sizes present sometimes a rate of mortality which is twice or three times the overall rate. The pattern is that of a monotonic increase in the level of mortality with family size. Since the average time-exposures are similar, the time location of the estimates by family size must be roughly comparable, so these ratios would not be seriously distorted by trends in mortality.

The level of mortality in Guatemala is lower than that in Bolivia. However, the pattern of variation by family size is strikingly similar. The differences in the relative risks by family size between the two countries are minimal for any family size by age groups.

The enormous differentials by family size observed in the two countries cannot be attributed entirely to the effects of birth order and concentration. Higher parities are strongly correlated with variables such as education and place of residence and the effects of the reproductive patterns cannot be assessed without controlling for those factors. However, there is little doubt that some positive correlation between the level of child mortality and the family size would remain after controlling for other factors.

In the case of these two countries respiratory diseases and enteritis and diarrhoea are very important causes of infant death, and the effect of birth order on the mortality rates from these causes surely play an important role in those differentials. As Papavangelou's analysis showed (Papavangelou, 1971), the mortality risks for a child born after a succession of births to relatively young mothers can be heightened not only because of factors directly linked to short birth intervals, like early weaning and maternal depletion, but other factors also play an important role. In Papavangelou's results the risk of infant death from enteritis and diarrhoea, for birth orders higher than six, was five times that observed for second births. Respiratory diseases had a much more severe impact on mortality rates for higher that for lower birth orders. The risk of deaths from accidents increased steadily with birth concentration. As commented in Chapter 2, these patterns appear to be related to increased oportunities for catching infections in an environment of poor sanitation as the family size increases, and diminished quality of maternal care when the mother has to give attention to several young children in the family.

Whatever the reasons, the observed differentials in child mortality by family size in these two countries are too dramatic by any standards. The rate for higher orders reaches in some cases a level which is between eight and ten fold that experienced for the lower birth orders in the same age group of mothers.

7.5 Conclusions

The results from models representing a range of nuptiality and fertility patterns have shown that the time-exposure to risk varies very little with family size for any age group of women. Data from four countries were analysed and similar patterns were encountered, corroborating the conclusions drawn from the model simulated results.

The pattern of constant exposures by family size is less clear in the case of mothers aged 15-19 and 20-24, and in one child families for women over forty. In the case of one child families the evidence suggests that the time-exposure may be longer than that for bigger families. If that is the case, the differential in mortality would be obscured since a lower risk would be offset by a longer exposure. In the data from the two countries analysed in the previous section there is no evidence that this might have happened. Mortality risks increased monotonically with family size for any age group of the mothers.

The mortality differential by family size may appear exagerated when a constant exposure is assumed for the age group of mothers 15-19 and 20-24. Comparing the "standard" with the "simulated" proportions obtained from the models, it is clear that the dominant factor, even in the case of younger mothers, is the differential mortality associated with the reproductive patterns. This is an important conclusion as it implies that the assumption of constant time-exposure by family size is quite safe and would not introduce a serious bias in the analysis of mortality by family size. Refinements in the methodology in order to allow for variable time exposures by family size within a given age group of mothers are possible, but they do not seem justified. It is unlikely that such efforts would lead to any rewarding conclusion: the cases in which variations in the time exposure may be relevant cover only a small proportion of births, and data errors may be as important as those arising from the simplifying assumptions.

The analyses of the data from Bolivia and Guatemala showed alarmingly strong mortality differentials by family size. As this is a univariate analysis, no definite conclusion can be drawn, but there is litle doubt that, in an environment of poor sanitation, factors associated to the number of children in the household increase the risk of mortality from respiratory diseases and from enteritis and diarrhoea, which are the most important causes of infant death in these countries, and that may explain part of that enormous differential.

APPENDIX 1

Computer Program to Estimate Mean Time-Exposures, "Standard," and "Simulated" Proportions of Children Surving

APENDIX 1

Computer program to estimate mean time-exposures, "standard" and "simulated" proportions of surviving children.

The program also provides some adictional estimates of fertility to help in the analysis, and a table of coefficients C_i to adjust estimates of mortality from age groups 15-19 and 20-24, basically.

```
      PROGRAM FINAL
C
C
C  2 DIMENSION ARRAYS : (I,J)
C  3 DIMENSION ARRAYS : (J,I,L)
C    J = BIRTH ORDER
C    I = MARRIAGE DURATION OR AGE OF THE WOMEN
C    L = NUMBER OF CHILDREN ATTAINED AT AGE (I)
C        *(PARITY)*
C
C
      DIMENSION PROB(20,15),SUM(40,15),SUMDIS(40,15),XNUFCI(40),
     1FECAGE(40,15),AGACUM(40,15),RISK(40,15),RISKEX(40,15),CONT(15),
     2DURINP(40,15),EQRISK(15,40,15),EXT(15,40,15),AGEBIR(15,40,15),
     3ANONLY(14,39,14),AGEADJ(14,39,14),FINRIS(14,39,14),STAND(0:40),
     4  AVPROP(39,14),SDPROP(39,14),SDLXS1(14,39,14),PROP(14,39,14),
     *     SIZWEG(39,14),AVSZ1(39),SURV(8),WEIGHT(0:4),AGACAA(40,15),
     *  AVSDPR(39),ALPHA5(7),SDSRV(8),TABLE(14,49),S(14),
     * AVPR5(8,10),SDPR5(8,10),TIME5(8,10),
     * PARACU(39),PARITY(8),YSD(7),ADSURV(7),XKADJ(7)
C
      EQUIVALENCE(EQRISK,ANONLY,SDLXS1)
      EQUIVALENCE(EXT,AGEADJ,PROP)
      EQUIVALENCE(AGEBIR,FINRIS)
C
C
      DATA WEIGHT/1.0,0.98020,0.96079,0.94176,0.92312/
      DATA STAND/ 1.0,.8499,.8070,.7876,.7762,.7691,.7642,.7601,
     *.7564,.7532,.7502,.7477,.7452,.7425,.7396,.7362,.7328,.7287,.7241,
     *.7188,.7130,.7069,.7005,.6943,.6884,.6826,.6764,.6703,.6643,.6584,
     *.6525,.6466,.6405,.6345,.6284,.6223,.6160,.6097,.6032,.5966,.5898/
C
      ALAT=0
      BLAT=0
      GLAT=0
      HLAT=0
      READ(5,4) CONT
    4 FORMAT(15F5.3)
C
    2 CONTINUE
C
      DO 700 LL=1,49
C
    5 READ(5,10,END=777) A,B,G,H,X1,Z
C
C  IF FERTILITY EQUAL PREVIOUS RUN, JUMP TO CALCULATE NUPTIALITY
```

```
C
   10 FORMAT(6F10.3 )
      IF(A.EQ.ALAT.AND.B.EQ.BLAT) GO TO 111
      DO 25 J=1,15
      DO 20 I=1,20
      IF(I.EQ.1.AND.J.EQ.1) THEN
      PROB(I,J)=A/(A+B)
      ELSE IF (I.LT.J) THEN
      PROB(I,J)=.0
      ELSE IF (I.NE.1.AND.I.EQ.J) THEN
      PROB(I,J)=(A+(J-1))/(A+B+(J-1))*PROB(I-1,J-1)
      ELSE
      PROB(I,J)=PROB(I-1,J)*((B+(I-1-J))*(I-1))/((I-J)*(A+B+(I-1)))
      END IF
   20 CONTINUE
   25 CONTINUE
C
C PROBABILITIES ARE ACCUMULATED
C
      DO 40 J=1,15
      DO 30 I=2,20
   30 PROB(I,J)=PROB(I-1,J)+PROB(I,J)
   40 CONTINUE
C
C CONVERTING THE LENGTH OF INTERVAL INTO YEAR'S UNITS
C
      DO 45 J=1,15
      DO 41 I=1,21
   41 SUM(I,J)=.0
      DO 42 I=2,40,2
   42 IF(I.GE.2*J) SUM(I,J)=PROB(I/2,J)
C
C CALCULATING ODD YEARS BY INTERPOLATION
C
      DO 43 I=5,37,2
   43 IF(I.GT.2*(J+1))SUM(I,J)=SUM(I-1,J)+.5*(SUM(I+1,J)-SUM(I-1,J))-
     1    .0625*(SUM(I+3,J)-SUM(I+1,J)-SUM(I-1,J)+SUM(I-3,J))
C
      SUM(2*J+1,J)=SUM(2*J,J)+.5*(SUM(2*J+2,J)-SUM(2*J,J))-.0625*
     1             (SUM(2*J+4,J)-SUM(2*J+2,J)-SUM(2*J,J))
C
      SUM(2*J-1,J)=SUM(2*J,J)-SUM(2*J+2,J)/4.+.125*
     1             (SUM(2*J+2,J)-2.*SUM(2*J,J))
C
   45 SUM(39,J)=SUM(38,J)+(SUM(40,J)-SUM(36,J))/4.+.125*
     1             (SUM(40,J)-2.*SUM(38,J)+SUM(36,J))
C
C SUBTRACTING TO CALCULATE PROBABILITIES BY YEARS
C
      DO 55 J=1,15
      DO 50 I=40,2,-1
   50 SUMDIS(I,J)=SUM(I,J)-SUM(I-1,J)
   55 SUMDIS(1,J)=SUM(1,J)
C
```

```
C  IF FERTILITY AND NUPT. EQUAL PREVIOUS RUN, JUMP TO MORTALITY
C
  111 IF(A.EQ.ALAT.AND.B.EQ.BLAT.AND.G.EQ.GLAT.AND.H.EQ.HLAT) GO TO 222
C
C CALCULATING THE NUPTIALITY MODEL
C
      Q=1.-G
      XNUPCI(1)=G**(H+1.)
      DO 60 I=2,40
   60 XNUPCI(I)=XNUPCI(I-1)*Q*(H+(I-1))/(FLOAT(I-1))
C
C MULTIPLYING THE DURATION MODEL BY THE MODEL OF NUPTIALITY
C
      DO 75 J=1,15
      DO 70 I=1,40
      SUPROV =.0
      DO 65 K=1,I
      PROVKK =XNUPCI(K)*SUMDIS(I+1-K,J)
   65 SUPROV =SUPROV+PROVKK
   70 FECAGE(I,J)=SUPROV
   75 CONTINUE
      DO 85 J=1,15
      AGACUM(1,J)=FECAGE(1,J)
      DO 80 I=2,40
   80 AGACUM(I,J)=AGACUM(I-1,J)+FECAGE(I,J)
   85 CONTINUE
C
C CALCULATING THE EXPOSURE TO THE RISK BY DURATION
C
      DO 100 J=1,15
      DO  95 I=1,40
      RISK(I,J)=.0
      SUMAGE=.0
      DO 90 K=1,I
   90 SUMAGE=SUMAGE+(I-K+.5)*SUMDIS(K,J)
   95 IF(SUM(I,J).GT..0) RISK(I,J)=SUMAGE/SUM(I,J)
  100 CONTINUE
C
C CALCULATING THE EXPOSURE TO THE RISK BY AGE OF THE MOTHERS
C
      DO 115 J=1,15
      DO 110 I=1,40
      RISKEX(I,J)=.0
      SUMAGE=.0
      DO 105 K=1,I
  105 SUMAGE=SUMAGE+(I-K+.5)*FECAGE(K,J)
  110 IF(AGACUM(I,J).GT..0) RISKEX(I,J)=SUMAGE/AGACUM(I,J)
  115 CONTINUE
C
C ** CALCULATING CONDITIONAL EXPOSURES FOR A GIVEN PARITY BY BIRTH ORDER
```

```
C
C  =TO LOCATE THE (N) DURATION EQUIVALENT IN EXPOSURE TO  AGE (I)
C
       DO 145 J=1,15
       DO 140 I=1,40
       IF(I.EQ.1.AND.J.EQ.1)THEN
          N=1
          GO TO 122
       END IF
       N=0
       DO 120 K=1,39
       IF(RISKEX(I,J).GT.0.AND.RISK(K,J).LE.RISKEX(I,J).AND.
     1    RISK(K+1,J).GT.RISKEX(I,J))  THEN
             N=K
             GO TO 122
       END IF
  120  CONTINUE
  122  CONTINUE
C
C  =INTERPOLATION TO GET EQUIVALENT RISKS IN ALL THE FOLLOWING ORDERS
C
       IF(N.NE.0) THEN
       DURINP(I,J)=FLOAT(N)+(RISKEX(I,J)-RISK(N,J))/(RISK(N+1,J) -
     1               RISK(N,J))+1.0
       ELSE
       DURINP(I,J)=.0
       END IF
       DO 125 L=1,15
       IF(L.LT.J.OR.N.EQ.0) THEN
       EQRISK(J,I,L)=.0
       ELSE
       EQRISK(J,I,L)=RISK(N,L)+(RISK(N+1,L)-RISK(N,L))/(RISK(N+1,J)-
     1                  RISK(N,J))*(RISKEX(I,J)-RISK(N,J))
       END IF
  125  CONTINUE
C
C  =CALCULATE THE EXTENSION-BACK FOR HIGHER BIRTH ORDERS
C
       DO 130 L=1,15
       IF(I.GT.2*(L-1).AND.L.GE.J) THEN
       EXT(J,I,L)=RISKEX(I,L)-EQRISK(J,I,L)
       ELSE
       EXT(J,I,L)=.0
       END IF
  130  CONTINUE
C
C  =AGE AT BIRTH OF J-TH CHILD FOR WOMEN OF PARITY "L"
C
       DO 135 L=1,15
       IF(L.EQ.1) THEN
```

```
      IF(J.EQ.1) THEN
      AGEBIR(J,I,L)=((DURINP(I,J)-EQRISK(J,I,L))*(FLOAT(J)/FLOAT(L)))
     *  - EXT(J,I,L)
      ELSE
      AGEBIR(J,I,L)=.0
      END IF
      ELSE IF(I.GT.2*(L-1).AND.L.GE.J.AND.EXT(J,I,L).GE.EXT(J,I,L-1))
     1 THEN
      AGEBIR(J,I,L)=((DURINP(I,J)-EQRISK(J,I,L))*(FLOAT(J)/FLOAT(L)))
     *  - EXT(J,I,L)
      ELSE
      AGEBIR(J,I,L)=.0
      END IF
  135 CONTINUE
  140 CONTINUE
  145 CONTINUE
C
C  =ADJUSTING FOR WOMEN WITH EXACTLY (N) CHILDREN AT AGE (I)
C
      DO 148 J=1,14
      DO 147 I=1,39
      DO 146 L=1,14
      IF(I.GT.2*(L-1).AND.L.GE.J) THEN
      IF(AGEBIR(J,I,L+1).GT..0) THEN
      ANONLY(J,I,L)=(AGEBIR(J,I,L)*AGACUM(I,L)-AGEBIR(J,I,L+1)*AGACUM
     1   (I,L+1))/(AGACUM(I,L)-AGACUM(I,L+1))
      ELSE
      ANONLY(J,I,L)=AGEBIR(J,I,L)
      END IF
      ELSE
      ANONLY(J,I,L)=.0
      END IF
  146 CONTINUE
  147 CONTINUE
  148 CONTINUE
C
      DO 150 L=1,14
      DO 149 J=1,L
      KK=2*J+1
      DO 149 I=KK,39
  149 IF(ANONLY(J,I,L).LT.ANONLY(J,I-1,L))ANONLY(J,I,L)=ANONLY(J,I-1,L)
  150 CONTINUE
C
C  =ADJUSTING TO TAKE INTO ACCOUNT THE STOPPING RULE
C
      DO 165 J=1,14
      DO 160 I=1,39
      DO 155 L=1,14
      IF(I.GT.2*(L-1).AND.L.GE.J) THEN
      AGEADJ(J,I,L)=CONT(L+1)/CONT(L)*ANONLY(J,I,L)+(1.0-CONT(L+1)/
```

```
     1  CONT(L))*AGEBIR(J,I,L)
        ELSE
        AGEADJ(J,I,L)=.0
        END IF
  155 CONTINUE
  160 CONTINUE
  165 CONTINUE
C
      DO 180 J=1,14
      DO 175 I=1,39
      DO 170 L=1,14
      IF(AGEADJ(J,I,L).GT..0) THEN
      FINRIS(J,I,L)=DURINP(I,J)-AGEADJ(J,I,L)
      ELSE
      FINRIS(J,I,L)=.0
      END IF
  170 CONTINUE
  175 CONTINUE
  180 CONTINUE
C
C
      DO 193 L=3,14
         K=2*L-2
      DO 192 I=K,39
      DO 191 J=2,L
  191 IF(FINRIS(J-1,I,L).LT.FINRIS(J,I,L))FINRIS(J,I,L)=0.0
  192 CONTINUE
  193 CONTINUE
C
C  STAND. PROP. OF SURV. CHILDREN ACCORDING TO AVERAGE TIME EXPOSURE
C
      DO 220 J=1,14
      DO 215 I=1,39
      DO 210 L=1,14
         T=FINRIS(J,I,L)
      IF (T.GT..0) THEN
        IF(T.LT.(1./12.)) THEN
          SDLXS1(J,I,L)=(1.-.07*T*12.)
        ELSE IF(T.GE.(1./12.).AND.T.LT..25) THEN
          SDLXS1(J,I,L)=0.93-0.02*6.*(T-(1./12.))
        ELSE IF(T.GE..25.AND.T.LT..5) THEN
          SDLXS1(J,I,L)=0.91-0.024*(T-.25)*4.
        ELSE IF(T.GE..5.AND.T.LT.1.) THEN
          SDLXS1(J,I,L)=0.886-.0361*(T-.5)*2.
        ELSE
          DO 202 K=1,I
          IF (T.GE.K-1.AND.T.LT.K) SDLXS1(J,I,L)=
     *    STAND(K-1)+(STAND(K)-STAND(K-1))*(T-(K-1))
  202     CONTINUE
        END IF
```

```
         ELSE
           SDLXS1(J,I,L)=.0
         END IF
  210 CONTINUE
  215 CONTINUE
  220 CONTINUE
C
  222 CONTINUE
C
C  OBTAINING THE EFFECTS BY ORDER, CONCENTRATION, AND MOTHER'S AGE
C
      DO 320 J=1,14
         XJ=J
      IF(J.LE.10) THEN
         AB=1.247-0.312*XJ+0.0817*XJ**2-0.0045*XJ**3
      ELSE
         AB=1.90
      END IF
C
      DO 315 I=1,39
          ED=X1+(I-1)
      DO 310 L=1,14
         T=FINRIS(J,I,L)
         AG=ED-T
      IF(AG.LT.20) THEN
         IF(J.EQ.1) K=3
         IF(J.EQ.2) K=6
         IF(J.EQ.3) K=8
         IF(J.EQ.4) K=9
         IF (J.GE.5) K=10
      ELSE IF (AG.GE.20.AND.AG.LT.25) THEN
         IF (J.EQ.1) K=3
         IF (J.EQ.2) K=5
         IF (J.EQ.3) K=6
         IF (J.EQ.4) K=7
         IF (J.EQ.5) K=9
         IF (J.GT.5) K=10
      ELSE IF (AG.GE.25.AND.AG.LT.30) THEN
         IF (J.LT.9) K=J+1
         IF ( J.GE.9) K=10
      ELSE IF (AG.GE.30.AND.AG.LT.35) THEN
         IF (J.LE.5) K=J
         IF (J.GT.5) K=J-1
      ELSE IF (AG.GE.35.AND.AG.LT.40) THEN
         IF (J.LE.3) K=J
         IF (J.GE.4.AND.J.LE.5) K=J-1
         IF (J.GE.6.AND.J.LE.7) K=4
         IF (J.GE.8.AND.J.LE.10) K=5
         IF(J.GE.11) K=6
      ELSE
```

```
            IF (J.EQ.1) K=1
            IF (J.GE.2.AND.J.LT.5) K=2
            IF (J.GE.5.AND.J.LE.10) K=3
            IF (J.GT.10) K=5
         END IF
C
         XAGE=(AG-12.0)/5.0
         XK=K
         AA=1.96-0.8109*XAGE+0.1725*XAGE**2-0.00944*XAGE**3
         IF (K.LE.10) THEN
         AC=1.18-0.31636*XK+0.07967*XK**2-0.003973*XK**3
         ELSE
         AC=2.1
         END IF
C
+C   "SIMULATED" PROPORTIONS OF SURVIVING CHILDREN
C
         IF(T.GT.0)THEN
           E=AA*AB*AC
         IF(T.GE.4)THEN
           E1=(1.+.75*(AB-1.))*(1.+.75*(AC-1.))
           E2=(1.+.50*(AB-1.))*(1.+.50*(AC-1.))
           E3=(1.+.25*(AB-1.))*(1.+.25*(AC-1.))
           Q0=.1501*E
           Q1=.0505*E1
           Q2=.0240*E2
           Q3=.0145*E3
           PR=(1.-Q0)*(1.-Q1)*(1.Q2)*(1.-Q3)*SDLXS1(J,I,L)/.7762
         ELSE IF(T.GE.3.AND.T.LT.4)THEN
           E1=(1.+.75*(AB-1.))*(1.+.75*(AC-1.))
           E2=(1.+.50*(AB-1.))*(1.+.50*(AC-1.))
           E3=(1.+.25*(AB-1.))*(1.+.25*(AC-1.))
           Q0=.1501*E
           Q1=.0505*E1
           Q2=.0240*E2
           Q3=.0145*E3
           PR=(1.-Q0)*(1.-Q1)*(1.-Q2)*(1.-Q3*(T-3.))
         ELSE IF(T.GE.2.5.AND.T.LT.3)THEN
           E1=(1.+.75*(AB-1.))*(1.+.75*(AC-1.))
           E2=(1.+.50*(AB-1.))*(1.+.50*(AC-1.))
           Q0=.1501*E
           Q1=.0505*E1
           PR=(1.-Q0)*(1.-Q1)*(1.-.0132*E2)*(1.-(T-2.5)*2.*.011*E2)
         ELSE IF(T.GE.2.AND.T.LT.2.5)THEN
           E1=(1.+.75*(AB-1.))*(1.+.75*(AC-1.))
           E2=(1.+.50*(AB-1.))*(1.+.50*(AC-1.))
           Q0=.1501*E
           Q1=.0505*E1
           PR=(1.-Q0)*(1.-Q1)*(1.-.0132*(T-2.)*2.*E2)
         ELSE IF(T.GE.1.5.AND.T.LT.2)THEN
```

```
          E1=(1.+.75*(AB-1.))*(1.+.75*(AC-1.))
          Q0=.1501*E
          PR=(1.-Q0)*(1.-.0324*E1)*(1.-(T-1.5)*2.*.0187*E1)
        ELSE IF(T.GE.1.AND.T.LT.1.5)THEN
          E1=(1.+.75*(AB-1.))*(1.+.75*(AC-1.))
          Q0=.1501*E
          PR=(1.-Q0)*(1.-.0324*(T-1.)*2.*E1)
        ELSE IF(T.GE..5.AND.T.LT.1)THEN
          PR=(1.-.114*E)*(1.-.0407*(T-.5)*2.*E)
        ELSE IF(T.GE..25.AND.T.LT..5)THEN
          PR=(1.-.09*E)*(1.-.0264*(T-.25)*4.*E)
        ELSE IF(T.GE.(1./12.).AND.T.LT..25)THEN
          PR=(1.-.07*E)*(1.-(T-(1./12.))*6.*.0215*E)
        ELSE
          PR=1.-.07*T*12*E
        END IF
C
        ELSE
           PR=.0
        END IF
        PROP(J,I,L)=PR
  310 CONTINUE
  315 CONTINUE
  320 CONTINUE
C
C
      DO 340 L=1,14
      SIZ=L
      DO 335 I=1,39
      AVPROP(I,L)=.0
      SDPROP(I,L)=.0
      DO 330 J=1,L
      SDPROP(I,L)=SDPROP(I,L)+SDLXS1(J,I,L)
  330 AVPROP(I,L)=AVPROP(I,L)+PROP(J,I,L)
      SDPROP(I,L)=SDPROP(I,L)/SIZ
      AVPROP(I,L)=AVPROP(I,L)/SIZ
  335 CONTINUE
  340 CONTINUE
C
C
      DO 360 J=1,15
      W=CONT(J)/AGACUM(40,J)
      DO 350 I=1,40
  350 AGACAA(I,J)=AGACUM(I,J)*W
  360 CONTINUE
C
C
      DO 370 I=1,39
      PARACU(I)=.0
      DO 365 J=1,14
```

```
      IF(AVPROP(I,J).GT.0)THEN
           PARACU(I)=PARACU(I)+AGACAA(I,J)
         IF(AVPROP(I,J+1).GT.0)THEN
           SIZWEG(I,J)=AGACAA(I,J)-AGACAA(I,J+1)
         ELSE
           SIZWEG(I,J)=AGACAA(I,J)
         END IF
      ELSE
         SIZWEG(I,J)=0.0
      END IF
  365 CONTINUE
  370 CONTINUE
C
C
      DO 380 I=1,39
      SUMB=.0
      DENOM=.0
      SUMB2=.0
      DO 375 J=1,14
      WW=J*SIZWEG(I,J)
      SUMB=SUMB+AVPROP(I,J)*WW
      SUMB2=SUMB2+SDPROP(I,J)*WW
  375 DENOM=DENOM+WW
      AVSZ1(I)=SUMB/DENOM
      AVSDPR(I)=SUMB2/DENOM
  380 CONTINUE
C
C
      MIN=16-X1
      IF(MIN.LT.1)THEN
      M=1
      ELSE
      M=MIN
      END IF
      DO 390 I=1,8
      PARITY(I)=.0
      SDSRV(I)=.0
  390 SURV(I)=0.0
      IN=MIN+5
      IF  (MIN.LT.1)  THEN
      IK=1-MIN
      ELSE
      IK=0
      END IF
      SNUM=0.0
      SDNU=.0
      PARNUM=.0
      SDIV=0.0
      K=1
      DO 450 I=M,39
```

```
        IF(I.LT.IN) THEN
             SDIV=SDIV+WEIGHT(IK)
             PARNUM=PARNUM+PARACU(I)*WEIGHT(IK)
             SDNU=SDNU+AVSDPR(I)*WEIGHT(IK)
             SNUM=SNUM+AVSZ1(I)*WEIGHT(IK)
             IK=IK+1
          IF(I.EQ.39) THEN
             SURV(K)=SNUM/SDIV
             SDSRV(K)=SDNU/SDIV
             PARITY(K)=PARNUM/SDIV
          END IF
        ELSE
          IF(SDIV.NE..0) THEN
             PARITY(K)=PARNUM/SDIV
             SURV(K)=SNUM/SDIV
             SDSRV(K)=SDNU/SDIV
          END IF
           IN=IN+5.0
           PARNUM=PARACU(I)
           SDNU=AVSDPR(I)
           SNUM=AVSZ1(I)
           IK=1
          IF(AVSZ1(I).GT..0) THEN
             SDIV=1.0
          ELSE
             SDIV=0.0
          END IF
           K=K+1
        END IF
    450 CONTINUE
C
        WRITE(6,452)
    452 FORMAT( 1H1 /// 15X,'*** PARAMETERS OF FERTILITY ***'  //
       *   1X,'STOPPING RULE =' )
        WRITE(6,455) CONT,A,B,G,H,X1
    455 FORMAT(1X,8F7.3  / 1X,7F7.3 // 1X,'A=',F5.3,5X,'B=',F5.3,5X,
       * 'G=',F5.3,5X,'H=',F5.2,5X,'X1=',F5.2 )
C
        DO 480 I=1,7
        YSD(I)=0.5*(ALOG((1.0-SDSRV(I))/SDSRV(I)))
        ALPHA5(I)=0.5*(ALOG((1.-SURV(I))/SURV(I)))-YSD(I)
    480 CONTINUE
C
        DO 490 I=1,7
        ADSURV(I)=1.0/(1.0+EXP(2.0*(ALPHA5(6)+YSD(I))))
        XKADJ(I)=(1.0-ADSURV(I))/(1.0-SURV(I))
    490 CONTINUE
C
        PARA1=PARITY(1)/PARITY(2)
        PARA2=PARITY(2)/PARITY(3)
```

```
C
      TFR=CONT(1)
      DO 540 I=2,15
  540 TFR=TFR+CONT(I)
      XMEAN=((H+1.)*(1.0-G))/G
      VAR=XMEAN/G
      XXMEAN=0.5+XMEAN
C
      MIN=16-X1
      IF(MIN.LT.1)THEN
      M=1
      ELSE
      M=MIN
      END IF
      DO 580 J=1,10
      DO 560 I=1,8
      SDPR5(I,J)=.0
  560 AVPR5(I,J)=0.0
      IN=MIN+5
      IF (MIN.LT.1) THEN
      IK=1-MIN
      ELSE
      IK=0
      END IF
      SNUM=0.0
      SNUM1=.0
      SDIV=0.0
      K=1
      DO 570 I=M,39
      IF(I.LT.IN) THEN
      IF(SDPROP(I,J).GT..0) THEN
      SDIV=SDIV+WEIGHT(IK)
      SNUM=SNUM+SDPROP(I,J)*WEIGHT(IK)
      SNUM1=SNUM1+AVPROP(I,J)*WEIGHT(IK)
      IK=IK+1
      ELSE
      IK=IK+1
      END IF
      IF(I.EQ.39) SDPR5(K,J)=SNUM/SDIV
      IF(I.EQ.39) AVPR5(K,J)=SNUM1/SDIV
      ELSE
      IF(SDIV.NE..0) SDPR5(K,J)=SNUM/SDIV
      IF(SDIV.NE..0) AVPR5(K,J)=SNUM1/SDIV
      IN=IN+5.0
      SNUM=SDPROP(I,J)
      SNUM1=AVPROP(I,J)
      IK=1
      IF(SDPROP(I,J).GT..0) THEN
      SDIV=1.0
      ELSE
```

```
      SDIV=0.0
      END IF
      K=K+1
      END IF
  570 CONTINUE
  580 CONTINUE
C
C
      DO 600 J=1,10
      DO 590 I=1,8
      P=SDPR5(I,J)
      IF(P.GT..0) THEN
      IF(P.GE..93)THEN
        TIME5(I,J)=(1.-P)/.07/12.
      ELSE IF(P.LT..93.AND.P.GE..91) THEN
        TIME5(I,J)=(1./12.)+(.93-P)/.02/6.
      ELSE IF(P.LT..91.AND.P.GE..886) THEN
        TIME5(I,J)=0.25+(.91-P)/.024/4.
      ELSE IF(P.LT..886.AND.P.GE..8499) THEN
        TIME5(I,J)=0.5+(.886-P)/.0361/2.
      ELSE
        DO 585 K=1,35
  585   IF(P.LE.STAND(K).AND.P.GT.STAND(K+1))
     *  TIME5(I,J)=K+(STAND(K)-P)/(STAND(K)-STAND(K+1))
      END IF
      ELSE
        TIME5(I,J)=0.0
      END IF
  590 CONTINUE
  600 CONTINUE
C
      WRITE(6,640)
  640 FORMAT(/// 1X,'TIME EXPOSURE TO THE RISK OF DYING' //
     * 1X,'ORDER',3X,'15-19',3X,'20-24',3X,'25-29',3X,'30-34',3X,
     *    '35-39',3X,'40-44',3X,'45-49' /)
      WRITE(6,620)(J,(TIME5(I,J),I=1,7),J=1,10)
C
C
      WRITE(6,630)
  630 FORMAT(/// 1X,'STANDARD PROPORTIONS OF SURVIVING CHILDREN' //
     * 1X,'ORDER',3X,'15-19',3X,'20-24',3X,'25-29',3X,'30-34',3X,
     *    '35-39',3X,'40-44',3X,'45-49' /)
      WRITE(6,620)(J,(SDPR5(I,J),I=1,7),J=1,10)
C
      WRITE(6,610)
  610 FORMAT(///,1X,'SIMULATEDPROPORTIONS OF SURVIVING CHILDREN' //
     * 1X,'ORDER',3X,'15-19',3X,'20-24',3X,'25-29',3X,'30-34',3X,
     *    '35-39',3X,'40-44',3X,'45-49' /)
      WRITE(6,620)(J,(AVPR5(I,J),I=1,7),J=1,10)
  620 FORMAT(2X,I2,2X,7F8.3)
```

```
C
C
      TABLE(1,LL)=PARA1
      TABLE(2,LL)=PARA2
      TABLE(3,LL)=XXMEAN
      TABLE(4,LL)=XXMEAN+X1
      TABLE(5,LL)=VAR
      TABLE(6,LL)=TFR
      TABLE(14,LL)=A/(A+B)
      DO 650 K=1,7
      TABLE(K+6,LL)=XKADJ(K)
  650 CONTINUE
C
     ALAT=A
      BLAT=B
      GLAT=G
      HLAT=H
  700 CONTINUE
C
  777 CONTINUE
C
C       ORGANIZING THE OUTPUT TABLES
C
C
      DO 750 L=2,49
      DO 740 K=1,L-1
      IF(TABLE(1,L).GT.TABLE(1,K)) THEN
          DO 720 I=1,14
  720     S(I)=TABLE(I,L)
          DO 730 M=L-1,K,-1
          DO 725 I=1,14
  725     TABLE(I,M+1)=TABLE(I,M)
  730     CONTINUE
          DO 735 I=1,14
  735     TABLE(I,K)=S(I)
          GO TO 750
      END IF
  740 CONTINUE
  750 CONTINUE
C
C
      PRINT 770
  770 FORMAT(1H1 // 30X,'*** TABLE OF PARAMETERS AND MULTIPLIERS ***'
     *    34X, '************************************'   /
     *  1H0,'    PAR1    PAR2  XXMEAN    AGE     VAR     TFR    15-19  20-24
     *25-29  30-34  35-39  40-44  45-49     P' )
      WRITE(6,780) TABLE
  780 FORMAT(1X,2X,F5.3,2X,F5.3,2X,F5.2,2X,F5.2,2X,F5.2,2X,F5.2,
     * 2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.2,2X,F5.3,3X,F4.3
C
      IF(Z.EQ.0) GO TO 2
C
      STOP
      END
```

APPENDIX 2

Model Time-Exposures and Proportions Surviving for Different Patterns of Nuptiality and Fertility

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =

0.900	0.860	0.790	0.700	0.560	0.430	0.300	0.210
0.120	0.060	0.030	0.020	0.010	0.006	0.004	

A=5.500 B=1.500 G=0.530 H= 7.00 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.46	2.32	3.84	7.79	12.92	17.96	22.79
2	1.76	2.68	3.82	5.94	9.91	15.33	20.20
3	2.02	2.83	4.07	5.91	8.67	13.33	18.40
4	0.0	3.09	4.41	6.47	9.19	13.27	17.51
5	0.0	3.55	4.41	6.59	9.41	13.49	17.30
6	0.0	0.0	4.80	6.76	9.88	14.15	17.72
7	0.0	0.0	0.0	6.55	9.76	14.13	17.64
8	0.0	0.0	0.0	7.27	10.25	14.81	18.27
9	0.0	0.0	0.0	7.79	10.23	14.97	18.44
10	0.0	0.0	0.0	0.0	9.89	14.76	18.29

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.830	0.801	0.778	0.757	0.743	0.724	0.696
2	0.817	0.794	0.778	0.764	0.750	0.735	0.712
3	0.807	0.791	0.776	0.765	0.754	0.742	0.722
4	0.0	0.787	0.773	0.762	0.753	0.742	0.726
5	0.0	0.781	0.773	0.762	0.752	0.741	0.727
6	0.0	0.0	0.771	0.761	0.751	0.739	0.725
7	0.0	0.0	0.0	0.762	0.751	0.739	0.726
8	0.0	0.0	0.0	0.759	0.750	0.737	0.723
9	0.0	0.0	0.0	0.757	0.750	0.736	0.722
10	0.0	0.0	0.0	0.0	0.751	0.737	0.723

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.790	0.815	0.815	0.798	0.783	0.763	0.733
2	0.738	0.778	0.803	0.815	0.802	0.786	0.761
3	0.677	0.737	0.783	0.804	0.805	0.791	0.770
4	0.0	0.703	0.749	0.780	0.788	0.784	0.767
5	0.0	0.680	0.713	0.755	0.764	0.766	0.754
6	0.0	0.0	0.689	0.720	0.739	0.741	0.735
7	0.0	0.0	0.0	0.683	0.717	0.724	0.715
8	0.0	0.0	0.0	0.653	0.677	0.691	0.695
9	0.0	0.0	0.0	0.626	0.642	0.665	0.669
10	0.0	0.0	0.0	0.0	0.613	0.645	0.654

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =

0.900	0.860	0.790	0.700	0.560	0.430	0.300	0.210
0.120	0.060	0.030	0.020	0.010	0.006	0.004	

A=4.500   B=2.500   G=0.530   H= 7.00   X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.57	2.67	4.22	6.96	11.46	16.55	21.42
2	1.78	2.85	4.27	6.43	9.10	13.15	17.46
3	1.94	2.90	4.39	6.47	9.12	12.88	16.73
4	0.0	3.10	4.52	6.78	9.73	13.82	17.35
5	0.0	3.55	4.43	6.75	9.85	14.12	17.57
6	0.0	0.0	4.58	6.77	10.10	14.52	17.98
7	0.0	0.0	4.84	6.49	9.86	14.41	17.89
8	0.0	0.0	0.0	6.78	10.12	14.83	18.30
9	0.0	0.0	0.0	7.24	9.98	14.86	18.37
10	0.0	0.0	0.0	0.0	9.65	14.61	18.21

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.825	0.794	0.775	0.760	0.747	0.731	0.704
2	0.817	0.791	0.774	0.762	0.753	0.742	0.727
3	0.810	0.790	0.773	0.762	0.753	0.743	0.730
4	0.0	0.786	0.772	0.761	0.751	0.740	0.727
5	0.0	0.781	0.773	0.761	0.751	0.739	0.726
6	0.0	0.0	0.772	0.761	0.750	0.738	0.724
7	0.0	0.0	0.770	0.762	0.751	0.738	0.725
8	0.0	0.0	0.0	0.761	0.750	0.737	0.723
9	0.0	0.0	0.0	0.759	0.750	0.737	0.722
10	0.0	0.0	0.0	0.0	0.751	0.738	0.723

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.786	0.806	0.810	0.804	0.791	0.775	0.747
2	0.737	0.773	0.795	0.810	0.803	0.791	0.773
3	0.696	0.735	0.778	0.796	0.801	0.789	0.771
4	0.0	0.703	0.746	0.774	0.782	0.781	0.764
5	0.0	0.680	0.713	0.751	0.761	0.759	0.752
6	0.0	0.0	0.677	0.719	0.738	0.738	0.729
7	0.0	0.0	0.648	0.684	0.717	0.721	0.712
8	0.0	0.0	0.0	0.648	0.679	0.691	0.693
9	0.0	0.0	0.0	0.620	0.642	0.666	0.671
10	0.0	0.0	0.0	0.0	0.614	0.644	0.654

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.900 0.860 0.790 0.700 0.560 0.430 0.300 0.210
0.120 0.060 0.030 0.020 0.010 0.006 0.004

A=4.000 B=3.000 G=0.530 H= 7.00 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.61	2.79	4.46	6.92	10.33	14.89	19.85
2	1.78	2.90	4.45	6.68	9.22	12.88	16.67
3	1.94	2.92	4.49	6.65	9.38	13.14	16.77
4	0.0	2.97	4.55	6.86	9.88	14.04	17.45
5	0.0	3.42	4.43	6.77	9.93	14.25	17.67
6	0.0	0.0	4.57	6.75	10.10	14.57	18.00
7	0.0	0.0	4.82	6.46	9.84	14.42	17.91
8	0.0	0.0	0.0	6.43	10.03	14.76	18.24
9	0.0	0.0	0.0	6.85	9.88	14.76	18.29
10	0.0	0.0	0.0	0.0	9.55	14.52	18.14

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.824	0.792	0.773	0.760	0.749	0.737	0.714
2	0.816	0.789	0.773	0.761	0.753	0.743	0.730
3	0.810	0.789	0.773	0.762	0.752	0.742	0.730
4	0.0	0.788	0.772	0.761	0.751	0.739	0.727
5	0.0	0.783	0.773	0.761	0.750	0.739	0.726
6	0.0	0.0	0.772	0.761	0.750	0.738	0.724
7	0.0	0.0	0.770	0.762	0.751	0.738	0.725
8	0.0	0.0	0.0	0.762	0.750	0.737	0.723
9	0.0	0.0	0.0	0.761	0.751	0.737	0.723
10	0.0	0.0	0.0	0.0	0.752	0.738	0.723

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.784	0.802	0.807	0.804	0.796	0.783	0.759
2	0.737	0.768	0.793	0.808	0.802	0.791	0.774
3	0.696	0.734	0.777	0.791	0.800	0.788	0.770
4	0.0	0.697	0.746	0.773	0.779	0.780	0.763
5	0.0	0.670	0.713	0.749	0.760	0.756	0.751
6	0.0	0.0	0.677	0.719	0.736	0.738	0.727
7	0.0	0.0	0.649	0.684	0.715	0.719	0.711
8	0.0	0.0	0.0	0.643	0.679	0.690	0.693
9	0.0	0.0	0.0	0.615	0.642	0.667	0.672
10	0.0	0.0	0.0	0.0	0.614	0.646	0.655

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =

0.900 0.860 0.790 0.700 0.560 0.430 0.300 0.210
0.120 0.060 0.030 0.020 0.010 0.006 0.004

A=5.000 B=2.000 G=0.520 H= 6.00 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.59	2.67	4.37	8.07	13.21	18.22	23.07
2	1.81	2.88	4.30	6.61	9.90	15.13	19.95
3	1.96	2.95	4.47	6.57	9.36	13.52	17.71
4	0.0	3.04	4.67	6.98	9.97	14.15	17.79
5	0.0	3.51	4.59	6.99	10.17	14.42	17.89
6	0.0	0.0	4.74	7.07	10.58	14.90	18.29
7	0.0	0.0	4.97	6.79	10.35	14.82	18.20
8	0.0	0.0	0.0	6.79	10.74	15.34	18.68
9	0.0	0.0	0.0	7.25	10.64	15.41	18.79
10	0.0	0.0	0.0	7.65	10.20	15.16	18.61

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.824	0.794	0.774	0.756	0.742	0.723	0.694
2	0.815	0.790	0.774	0.762	0.750	0.736	0.713
3	0.809	0.789	0.773	0.762	0.752	0.741	0.725
4	0.0	0.787	0.771	0.760	0.750	0.739	0.725
5	0.0	0.782	0.772	0.760	0.750	0.738	0.725
6	0.0	0.0	0.771	0.760	0.749	0.737	0.723
7	0.0	0.0	0.769	0.761	0.749	0.737	0.723
8	0.0	0.0	0.0	0.761	0.748	0.735	0.720
9	0.0	0.0	0.0	0.759	0.749	0.735	0.720
10	0.0	0.0	0.0	0.758	0.750	0.736	0.721

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.784	0.806	0.808	0.796	0.781	0.761	0.731
2	0.734	0.769	0.795	0.809	0.800	0.785	0.761
3	0.693	0.733	0.777	0.795	0.801	0.788	0.770
4	0.0	0.694	0.744	0.772	0.779	0.779	0.763
5	0.0	0.666	0.707	0.748	0.760	0.757	0.750
6	0.0	0.0	0.670	0.715	0.734	0.736	0.727
7	0.0	0.0	0.645	0.681	0.707	0.718	0.710
8	0.0	0.0	0.0	0.639	0.669	0.686	0.689
9	0.0	0.0	0.0	0.611	0.637	0.657	0.666
10	0.0	0.0	0.0	0.579	0.607	0.640	0.651

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.900 0.860 0.790 0.700 0.560 0.430 0.300 0.210
0.120 0.060 0.030 0.020 0.010 0.006 0.004

A=4.500 B=2.500 G=0.560 H= 5.50 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.77	3.24	5.58	9.42	14.52	19.50	24.43
2	1.88	3.27	5.13	7.80	11.25	15.61	19.61
3	2.02	3.22	5.01	7.61	10.91	15.00	18.40
4	2.48	3.22	5.10	7.84	11.53	15.61	18.79
5	0.0	3.51	4.91	7.70	11.60	15.78	18.88
6	0.0	3.76	4.78	7.67	11.82	16.09	19.16
7	0.0	0.0	4.99	7.32	11.49	15.93	19.03
8	0.0	0.0	5.43	7.23	11.74	16.26	19.38
9	0.0	0.0	0.0	7.31	11.57	16.24	19.41
10	0.0	0.0	0.0	7.73	11.04	15.98	19.22

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.817	0.785	0.766	0.752	0.738	0.716	0.686
2	0.812	0.785	0.768	0.757	0.747	0.734	0.715
3	0.807	0.785	0.769	0.758	0.748	0.736	0.722
4	0.798	0.785	0.769	0.757	0.746	0.734	0.720
5	0.0	0.782	0.770	0.758	0.746	0.734	0.719
6	0.0	0.779	0.771	0.758	0.746	0.732	0.718
7	0.0	0.0	0.769	0.759	0.746	0.733	0.719
8	0.0	0.0	0.767	0.759	0.746	0.732	0.717
9	0.0	0.0	0.0	0.759	0.746	0.732	0.716
10	0.0	0.0	0.0	0.757	0.748	0.733	0.718

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.775	0.791	0.794	0.786	0.771	0.749	0.717
2	0.728	0.759	0.780	0.800	0.793	0.781	0.761
3	0.688	0.725	0.762	0.782	0.790	0.781	0.763
4	0.660	0.684	0.733	0.761	0.770	0.767	0.756
5	0.0	0.655	0.700	0.734	0.747	0.747	0.735
6	0.0	0.625	0.657	0.706	0.723	0.727	0.718
7	0.0	0.0	0.632	0.668	0.696	0.706	0.702
8	0.0	0.0	0.600	0.628	0.661	0.676	0.678
9	0.0	0.0	0.0	0.596	0.628	0.645	0.656
10	0.0	0.0	0.0	0.571	0.596	0.627	0.636

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.900 0.860 0.790 0.700 0.560 0.430 0.300 0.210
0.120 0.060 0.030 0.020 0.010 0.006 0.004

A=4.000 B=3.000 G=0.560 H= 5.50 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.81	3.39	5.63	8.75	12.84	17.67	22.51
2	1.89	3.35	5.29	7.87	11.01	14.91	18.39
3	2.02	3.25	5.11	7.72	11.03	14.99	18.22
4	2.48	3.22	5.11	7.87	11.62	15.66	18.75
5	0.0	3.50	4.90	7.69	11.63	15.81	18.87
6	0.0	3.75	4.76	7.63	11.78	16.06	19.11
7	0.0	0.0	4.79	7.28	11.43	15.88	18.99
8	0.0	0.0	5.40	7.15	11.62	16.16	19.28
9	0.0	0.0	0.0	7.24	11.42	16.13	19.30
10	0.0	0.0	0.0	7.66	10.91	15.86	19.12

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.815	0.783	0.766	0.754	0.743	0.726	0.697
2	0.812	0.784	0.768	0.757	0.748	0.737	0.722
3	0.807	0.785	0.769	0.757	0.748	0.736	0.723
4	0.798	0.785	0.769	0.757	0.746	0.734	0.720
5	0.0	0.782	0.770	0.758	0.746	0.733	0.719
6	0.0	0.779	0.771	0.758	0.746	0.733	0.718
7	0.0	0.0	0.771	0.759	0.747	0.733	0.719
8	0.0	0.0	0.767	0.760	0.746	0.732	0.717
9	0.0	0.0	0.0	0.759	0.747	0.732	0.717
10	0.0	0.0	0.0	0.758	0.748	0.733	0.718

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.773	0.789	0.794	0.791	0.783	0.766	0.736
2	0.728	0.757	0.778	0.799	0.794	0.783	0.766
3	0.688	0.724	0.761	0.780	0.788	0.780	0.763
4	0.660	0.684	0.733	0.760	0.769	0.766	0.755
5	0.0	0.655	0.700	0.733	0.747	0.746	0.734
6	0.0	0.625	0.658	0.706	0.724	0.726	0.718
7	0.0	0.0	0.629	0.671	0.695	0.705	0.702
8	0.0	0.0	0.600	0.628	0.661	0.677	0.678
9	0.0	0.0	0.0	0.599	0.628	0.647	0.656
10	0.0	0.0	0.0	0.572	0.596	0.627	0.639

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =

0.900	0.860	0.790	0.700	0.560	0.430	0.300	0.210
0.120	0.060	0.030	0.020	0.010	0.006	0.004	

A=3.500 B=3.500 G=0.600 H= 5.50 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.92	3.80	6.27	9.18	12.48	16.10	19.96
2	1.94	3.60	5.82	8.45	11.76	15.45	18.56
3	2.08	3.43	5.51	8.28	11.86	15.73	18.66
4	2.53	3.33	5.42	8.32	12.35	16.26	19.18
5	0.0	3.40	5.10	8.06	12.30	16.34	19.26
6	0.0	3.82	4.89	7.94	12.36	16.50	19.44
7	0.0	0.0	4.89	7.57	12.01	16.32	19.31
8	0.0	0.0	5.26	7.41	12.12	16.53	19.56
9	0.0	0.0	0.0	7.10	11.90	16.48	19.56
10	0.0	0.0	0.0	7.52	11.38	16.23	19.38

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.810	0.778	0.763	0.753	0.744	0.732	0.713
2	0.810	0.781	0.765	0.755	0.746	0.735	0.721
3	0.806	0.783	0.767	0.755	0.746	0.734	0.721
4	0.797	0.784	0.767	0.755	0.744	0.732	0.718
5	0.0	0.783	0.769	0.756	0.744	0.731	0.717
6	0.0	0.778	0.770	0.757	0.744	0.731	0.716
7	0.0	0.0	0.770	0.758	0.745	0.731	0.717
8	0.0	0.0	0.768	0.759	0.745	0.731	0.716
9	0.0	0.0	0.0	0.760	0.745	0.731	0.716
10	0.0	0.0	0.0	0.758	0.747	0.732	0.717

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.767	0.780	0.787	0.788	0.785	0.777	0.758
2	0.724	0.751	0.772	0.790	0.790	0.780	0.764
3	0.685	0.720	0.753	0.775	0.781	0.776	0.759
4	0.657	0.681	0.728	0.754	0.764	0.758	0.751
5	0.0	0.649	0.693	0.728	0.740	0.742	0.729
6	0.0	0.621	0.652	0.700	0.717	0.720	0.715
7	0.0	0.0	0.623	0.661	0.690	0.700	0.695
8	0.0	0.0	0.592	0.623	0.656	0.671	0.675
9	0.0	0.0	0.0	0.585	0.619	0.641	0.651
10	0.0	0.0	0.0	0.564	0.588	0.622	0.633

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.920 0.830 0.820 0.750 0.680 0.590 0.480 0.350
0.220 0.130 0.070 0.050 0.030 0.020 0.010

A=3.500 B=3.500 G=0.580 H= 5.50 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.88	3.66	5.99	8.85	12.13	15.69	19.55
2	1.92	3.49	5.57	8.14	11.28	14.98	18.21
3	2.05	3.32	5.25	7.89	11.23	15.16	18.24
4	2.50	3.21	4.99	7.62	11.24	15.27	18.32
5	0.0	3.50	4.84	7.51	11.32	15.51	18.55
6	0.0	3.76	4.71	7.43	11.42	15.73	18.78
7	0.0	0.0	4.82	7.33	11.55	15.98	19.03
8	0.0	0.0	5.17	7.15	11.61	16.13	19.21
9	0.0	0.0	0.0	6.89	11.34	16.03	19.17
10	0.0	0.0	0.0	7.37	11.00	15.92	19.13

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.812	0.780	0.764	0.754	0.745	0.734	0.716
2	0.811	0.782	0.766	0.756	0.747	0.736	0.723
3	0.806	0.784	0.768	0.757	0.747	0.736	0.723
4	0.797	0.785	0.769	0.758	0.747	0.735	0.722
5	0.0	0.782	0.770	0.758	0.747	0.734	0.721
6	0.0	0.779	0.771	0.759	0.747	0.734	0.720
7	0.0	0.0	0.770	0.759	0.746	0.733	0.719
8	0.0	0.0	0.768	0.760	0.746	0.732	0.718
9	0.0	0.0	0.0	0.761	0.747	0.733	0.718
10	0.0	0.0	0.0	0.759	0.748	0.733	0.718

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.769	0.783	0.789	0.790	0.788	0.779	0.761
2	0.726	0.754	0.775	0.796	0.792	0.782	0.765
3	0.686	0.722	0.756	0.778	0.787	0.778	0.762
4	0.659	0.684	0.733	0.761	0.769	0.767	0.755
5	0.0	0.655	0.700	0.734	0.746	0.747	0.738
6	0.0	0.626	0.655	0.706	0.724	0.726	0.719
7	0.0	0.0	0.627	0.669	0.694	0.704	0.699
8	0.0	0.0	0.595	0.627	0.663	0.675	0.679
9	0.0	0.0	0.0	0.595	0.626	0.650	0.657
10	0.0	0.0	0.0	0.569	0.593	0.630	0.638

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =

0.900	0.780	0.660	0.530	0.420	0.280	0.180	0.130
0.060	0.030	0.015	0.008	0.004	0.002	0.001	

A=3.500 B=3.500 G=0.560 H= 5.50 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.87	3.67	6.06	9.00	12.38	16.07	19.99
2	1.91	3.51	5.68	8.33	11.65	15.38	18.59
3	2.03	3.36	5.43	8.22	11.78	15.72	18.76
4	2.48	3.23	5.14	7.91	11.71	15.73	18.77
5	0.0	3.52	5.03	8.00	12.19	16.30	19.31
6	0.0	3.75	4.79	7.73	11.98	16.22	19.26
7	0.0	0.0	4.76	7.17	11.25	15.72	18.84
8	0.0	0.0	5.15	7.26	11.86	16.37	19.47
9	0.0	0.0	0.0	6.84	11.29	16.03	19.19
10	0.0	0.0	0.0	7.25	10.80	15.75	19.03

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.812	0.780	0.764	0.753	0.744	0.732	0.713
2	0.811	0.782	0.766	0.755	0.746	0.735	0.721
3	0.806	0.783	0.767	0.756	0.746	0.734	0.720
4	0.798	0.785	0.763	0.757	0.746	0.734	0.720
5	0.0	0.782	0.769	0.756	0.745	0.732	0.717
6	0.0	0.779	0.771	0.757	0.745	0.732	0.717
7	0.0	0.0	0.771	0.759	0.747	0.734	0.720
8	0.0	0.0	0.768	0.759	0.746	0.731	0.716
9	0.0	0.0	0.0	0.761	0.747	0.733	0.718
10	0.0	0.0	0.0	0.759	0.748	0.734	0.719

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.770	0.783	0.789	0.789	0.786	0.777	0.758
2	0.726	0.754	0.774	0.791	0.791	0.781	0.765
3	0.687	0.722	0.755	0.777	0.783	0.777	0.761
4	0.660	0.684	0.732	0.759	0.768	0.763	0.754
5	0.0	0.655	0.699	0.730	0.742	0.744	0.729
6	0.0	0.625	0.655	0.705	0.721	0.724	0.718
7	0.0	0.0	0.629	0.671	0.696	0.706	0.702
8	0.0	0.0	0.595	0.627	0.659	0.674	0.676
9	0.0	0.0	0.0	0.596	0.628	0.649	0.659
10	0.0	0.0	0.0	0.570	0.596	0.632	0.641

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.920 0.880 0.820 0.750 0.680 0.590 0.480 0.350
0.220 0.130 0.070 0.050 0.030 0.020 0.010

A=6.000 B=1.000 G=0.470 H= 7.00 X1=12.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.13	1.72	2.50	4.52	9.19	14.34	19.31
2	1.57	2.13	2.88	3.87	6.45	11.21	16.37
3	1.91	2.49	3.25	4.19	5.84	9.10	14.46
4	0.0	2.84	3.64	4.55	5.95	8.30	12.79
5	0.0	0.0	0.0	4.95	6.59	8.77	12.50
6	0.0	0.0	0.0	5.94	7.23	9.68	13.35
7	0.0	0.0	0.0	0.0	7.89	10.93	14.59
8	0.0	0.0	0.0	0.0	9.02	12.02	15.67
9	0.0	0.0	0.0	0.0	9.87	12.36	16.05
10	0.0	0.0	0.0	0.0	0.0	13.09	16.35

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.842	0.819	0.797	0.772	0.753	0.738	0.717
2	0.826	0.805	0.790	0.778	0.762	0.747	0.731
3	0.811	0.798	0.785	0.775	0.765	0.753	0.738
4	0.0	0.791	0.780	0.772	0.764	0.755	0.743
5	0.0	0.0	0.0	0.769	0.762	0.754	0.744
6	0.0	0.0	0.0	0.764	0.759	0.751	0.741
7	0.0	0.0	0.0	0.0	0.757	0.748	0.738
8	0.0	0.0	0.0	0.0	0.753	0.745	0.734
9	0.0	0.0	0.0	0.0	0.751	0.744	0.733
10	0.0	0.0	0.0	0.0	0.0	0.742	0.731

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.807	0.834	0.840	0.821	0.800	0.785	0.762
2	0.754	0.793	0.823	0.831	0.815	0.798	0.781
3	0.719	0.753	0.800	0.825	0.817	0.798	0.779
4	0.0	0.722	0.774	0.803	0.808	0.797	0.780
5	0.0	0.0	0.0	0.773	0.789	0.783	0.770
6	0.0	0.0	0.0	0.749	0.764	0.766	0.755
7	0.0	0.0	0.0	0.0	0.734	0.743	0.736
8	0.0	0.0	0.0	0.0	0.702	0.715	0.716
9	0.0	0.0	0.0	0.0	0.680	0.689	0.694
10	0.0	0.0	0.0	0.0	0.0	0.667	0.675

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.920 0.880 0.820 0.750 0.680 0.590 0.480 0.350
0.220 0.130 0.070 0.050 0.030 0.020 0.010

A=5.500 B=1.500 G=0.500 H= 8.00 X1=12.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.20	1.85	2.72	4.52	8.86	14.01	18.98
2	1.56	2.20	3.10	4.22	6.54	10.80	16.13
3	1.89	2.49	3.41	4.56	6.33	9.09	14.03
4	0.0	2.90	3.57	4.81	6.52	8.92	12.94
5	0.0	0.0	4.18	5.08	7.01	9.57	13.47
6	0.0	0.0	0.0	5.73	7.51	10.46	14.36
7	0.0	0.0	0.0	6.41	7.95	11.48	15.32
8	0.0	0.0	0.0	0.0	8.60	12.25	16.12
9	0.0	0.0	0.0	0.0	9.34	12.39	16.30
10	0.0	0.0	0.0	0.0	0.0	12.50	16.47

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.841	0.813	0.793	0.773	0.754	0.740	0.719
2	0.826	0.803	0.786	0.775	0.762	0.743	0.732
3	0.812	0.797	0.783	0.772	0.763	0.753	0.739
4	0.0	0.790	0.781	0.770	0.762	0.753	0.743
5	0.0	0.0	0.775	0.769	0.760	0.751	0.741
6	0.0	0.0	0.0	0.766	0.758	0.749	0.738
7	0.0	0.0	0.0	0.763	0.757	0.746	0.735
8	0.0	0.0	0.0	0.0	0.754	0.745	0.732
9	0.0	0.0	0.0	0.0	0.752	0.744	0.732
10	0.0	0.0	0.0	0.0	0.0	0.744	0.731

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.806	0.829	0.835	0.821	0.801	0.786	0.764
2	0.755	0.791	0.818	0.827	0.814	0.798	0.781
3	0.720	0.753	0.798	0.818	0.815	0.800	0.782
4	0.0	0.734	0.769	0.798	0.805	0.795	0.777
5	0.0	0.0	0.762	0.772	0.783	0.780	0.767
6	0.0	0.0	0.0	0.749	0.760	0.762	0.751
7	0.0	0.0	0.0	0.718	0.732	0.739	0.732
8	0.0	0.0	0.0	0.0	0.699	0.712	0.713
9	0.0	0.0	0.0	0.0	0.679	0.689	0.692
10	0.0	0.0	0.0	0.0	0.0	0.668	0.675

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.920 0.890 0.820 0.750 0.680 0.590 0.460 0.350
0.220 0.130 0.070 0.050 0.030 0.020 0.010

A=5.000 B=2.000 G=0.500 H= 8.00 X1=12.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.23	1.95	2.93	4.61	8.26	13.38	18.38
2	1.56	2.29	3.33	4.59	6.72	9.92	15.16
3	1.90	2.51	3.54	4.84	6.76	9.36	13.42
4	0.0	2.90	3.62	4.99	6.94	9.52	13.43
5	0.0	0.0	4.07	5.20	7.33	10.20	14.17
6	0.0	0.0	0.0	5.55	7.68	11.00	14.92
7	0.0	0.0	0.0	6.11	7.99	11.79	15.72
8	0.0	0.0	0.0	0.0	8.20	12.35	16.30
9	0.0	0.0	0.0	0.0	9.22	12.39	16.39
10	0.0	0.0	0.0	0.0	9.56	12.40	16.49

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.840	0.809	0.789	0.772	0.756	0.741	0.722
2	0.826	0.801	0.784	0.772	0.761	0.750	0.736
3	0.811	0.797	0.781	0.770	0.761	0.752	0.741
4	0.0	0.789	0.781	0.769	0.760	0.752	0.741
5	0.0	0.0	0.776	0.768	0.759	0.750	0.739
6	0.0	0.0	0.0	0.765	0.758	0.748	0.736
7	0.0	0.0	0.0	0.764	0.756	0.746	0.734
8	0.0	0.0	0.0	0.0	0.755	0.744	0.732
9	0.0	0.0	0.0	0.0	0.753	0.744	0.731
10	0.0	0.0	0.0	0.0	0.752	0.744	0.731

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.804	0.824	0.830	0.820	0.803	0.783	0.767
2	0.755	0.789	0.815	0.824	0.813	0.798	0.781
3	0.720	0.752	0.796	0.815	0.812	0.798	0.781
4	0.0	0.734	0.768	0.796	0.802	0.793	0.775
5	0.0	0.0	0.754	0.769	0.781	0.777	0.765
6	0.0	0.0	0.0	0.744	0.759	0.758	0.749
7	0.0	0.0	0.0	0.714	0.732	0.737	0.730
8	0.0	0.0	0.0	0.0	0.697	0.711	0.710
9	0.0	0.0	0.0	0.0	0.679	0.691	0.691
10	0.0	0.0	0.0	0.0	0.660	0.668	0.675

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.920 0.830 0.820 0.750 0.680 0.590 0.480 0.350
0.220 0.130 0.070 0.050 0.030 0.020 0.010

A=4.500 B=2.500 G=0.470 H= 7.70 X1=12.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.23	1.97	2.97	4.39	6.82	11.03	16.18
2	1.55	2.29	3.37	4.63	6.54	8.93	12.79
3	1.89	2.50	3.53	4.85	6.74	9.14	12.64
4	0.0	2.83	3.60	4.97	6.92	9.45	13.19
5	0.0	0.0	4.16	5.12	7.22	10.06	14.02
6	0.0	0.0	0.0	5.63	7.51	10.73	14.70
7	0.0	0.0	0.0	6.16	7.75	11.37	15.37
8	0.0	0.0	0.0	0.0	8.20	11.83	15.90
9	0.0	0.0	0.0	0.0	8.63	11.84	15.99
10	0.0	0.0	0.0	0.0	0.0	11.83	16.05

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.840	0.803	0.788	0.773	0.761	0.748	0.732
2	0.826	0.801	0.783	0.772	0.762	0.753	0.743
3	0.812	0.797	0.782	0.770	0.761	0.753	0.743
4	0.0	0.790	0.781	0.769	0.760	0.752	0.742
5	0.0	0.0	0.775	0.769	0.759	0.750	0.740
6	0.0	0.0	0.0	0.766	0.758	0.748	0.737
7	0.0	0.0	0.0	0.764	0.757	0.747	0.735
8	0.0	0.0	0.0	0.0	0.756	0.746	0.733
9	0.0	0.0	0.0	0.0	0.754	0.746	0.733
10	0.0	0.0	0.0	0.0	0.0	0.746	0.733

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.804	0.823	0.829	0.822	0.797	0.775	0.759
2	0.756	0.789	0.814	0.823	0.813	0.796	0.774
3	0.721	0.753	0.796	0.815	0.812	0.798	0.781
4	0.0	0.735	0.769	0.796	0.802	0.793	0.775
5	0.0	0.0	0.763	0.770	0.781	0.778	0.765
6	0.0	0.0	0.0	0.751	0.760	0.760	0.749
7	0.0	0.0	0.0	0.720	0.734	0.738	0.730
8	0.0	0.0	0.0	0.0	0.705	0.714	0.711
9	0.0	0.0	0.0	0.0	0.681	0.693	0.693
10	0.0	0.0	0.0	0.0	0.0	0.671	0.676

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.920 0.880 0.820 0.750 0.680 0.590 0.480 0.350
0.220 0.130 0.070 0.050 0.030 0.020 0.010

A=5.500 B=1.500 G=0.540 H= 6.00 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.57	2.62	4.73	9.47	14.63	19.61	24.56
2	1.81	2.83	4.20	6.99	11.84	17.08	21.75
3	1.97	2.93	4.33	6.43	9.66	14.91	19.69
4	0.0	3.00	4.38	6.33	9.15	13.59	17.91
5	0.0	3.51	4.45	6.57	9.43	13.72	17.52
6	0.0	0.0	4.67	6.77	9.86	14.23	17.77
7	0.0	0.0	5.07	6.90	10.44	14.85	18.26
8	0.0	0.0	0.0	6.91	10.86	15.38	18.70
9	0.0	0.0	0.0	7.36	10.74	15.40	18.74
10	0.0	0.0	0.0	7.90	10.55	15.41	18.79

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.825	0.795	0.771	0.752	0.737	0.715	0.685
2	0.815	0.791	0.775	0.760	0.746	0.728	0.702
3	0.808	0.789	0.774	0.762	0.751	0.737	0.715
4	0.0	0.788	0.773	0.763	0.753	0.741	0.725
5	0.0	0.782	0.773	0.762	0.752	0.740	0.726
6	0.0	0.0	0.771	0.761	0.751	0.739	0.725
7	0.0	0.0	0.769	0.761	0.749	0.737	0.723
8	0.0	0.0	0.0	0.760	0.748	0.735	0.720
9	0.0	0.0	0.0	0.759	0.748	0.735	0.720
10	0.0	0.0	0.0	0.757	0.749	0.735	0.720

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.785	0.807	0.803	0.785	0.771	0.747	0.716
2	0.734	0.774	0.797	0.808	0.794	0.776	0.748
3	0.692	0.733	0.779	0.797	0.801	0.785	0.762
4	0.0	0.695	0.751	0.779	0.787	0.783	0.764
5	0.0	0.665	0.711	0.752	0.764	0.762	0.753
6	0.0	0.0	0.670	0.718	0.740	0.740	0.731
7	0.0	0.0	0.642	0.677	0.707	0.719	0.710
8	0.0	0.0	0.0	0.635	0.669	0.686	0.689
9	0.0	0.0	0.0	0.606	0.636	0.657	0.666
10	0.0	0.0	0.0	0.576	0.604	0.631	0.648

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.920 0.880 0.820 0.750 0.680 0.590 0.480 0.350
0.220 0.130 0.070 0.050 0.030 0.020 0.010

A=5.000 B=2.000 G=0.600 H= 5.50 X1=11.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.81	3.48	6.75	11.76	16.84	21.70	26.69
2	1.91	3.36	5.45	8.63	13.43	18.48	22.95
3	2.07	3.30	5.14	7.90	11.67	16.14	19.77
4	2.53	3.24	4.93	7.60	11.27	15.49	18.80
5	0.0	3.55	4.90	7.59	11.43	15.67	18.79
6	0.0	3.82	4.81	7.63	11.71	15.99	19.03
7	0.0	0.0	4.94	7.64	12.05	16.35	19.38
8	0.0	0.0	5.33	7.51	12.25	16.63	19.68
9	0.0	0.0	0.0	7.19	12.01	16.54	19.64
10	0.0	0.0	0.0	7.66	11.68	16.45	19.61

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.815	0.782	0.761	0.746	0.729	0.702	0.672
2	0.811	0.783	0.767	0.754	0.741	0.722	0.695
3	0.806	0.784	0.768	0.757	0.746	0.732	0.714
4	0.797	0.785	0.769	0.753	0.747	0.735	0.720
5	0.0	0.781	0.770	0.758	0.747	0.734	0.720
6	0.0	0.778	0.770	0.758	0.746	0.733	0.719
7	0.0	0.0	0.770	0.758	0.745	0.731	0.717
8	0.0	0.0	0.767	0.758	0.745	0.730	0.715
9	0.0	0.0	0.0	0.759	0.745	0.731	0.715
10	0.0	0.0	0.0	0.758	0.746	0.731	0.715

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.773	0.787	0.781	0.766	0.749	0.722	0.691
2	0.726	0.757	0.777	0.790	0.784	0.763	0.735
3	0.685	0.723	0.761	0.780	0.787	0.776	0.757
4	0.657	0.683	0.734	0.765	0.770	0.766	0.755
5	0.0	0.653	0.699	0.734	0.747	0.747	0.735
6	0.0	0.621	0.653	0.705	0.722	0.726	0.719
7	0.0	0.0	0.622	0.662	0.691	0.703	0.697
8	0.0	0.0	0.591	0.622	0.657	0.670	0.673
9	0.0	0.0	0.0	0.584	0.621	0.642	0.650
10	0.0	0.0	0.0	0.560	0.584	0.619	0.630

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.900 0.780 0.660 0.530 0.420 0.280 0.180 0.130
0.060 0.030 0.015 0.008 0.004 0.002 0.001

A=6.000 B=1.000 G=0.530 H= 7.00 X1=12.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.32	2.09	3.73	7.80	12.93	17.96	22.79
2	1.62	2.47	3.64	5.81	10.19	15.47	20.37
3	1.96	2.69	3.89	5.75	8.79	14.04	19.00
4	0.0	2.97	3.97	5.79	8.31	12.52	17.72
5	0.0	0.0	4.22	6.42	9.22	13.28	17.60
6	0.0	0.0	4.92	6.38	9.30	13.37	17.32
7	0.0	0.0	0.0	6.22	8.72	12.65	16.52
8	0.0	0.0	0.0	7.34	9.84	14.45	18.13
9	0.0	0.0	0.0	0.0	9.45	14.10	17.75
10	0.0	0.0	0.0	0.0	10.10	13.94	17.61

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.836	0.805	0.779	0.757	0.743	0.724	0.696
2	0.823	0.798	0.780	0.765	0.750	0.735	0.711
3	0.809	0.794	0.777	0.765	0.754	0.739	0.719
4	0.0	0.788	0.777	0.765	0.755	0.744	0.725
5	0.0	0.0	0.775	0.762	0.753	0.742	0.726
6	0.0	0.0	0.770	0.763	0.752	0.741	0.727
7	0.0	0.0	0.0	0.763	0.754	0.743	0.731
8	0.0	0.0	0.0	0.759	0.751	0.738	0.723
9	0.0	0.0	0.0	0.0	0.752	0.739	0.725
10	0.0	0.0	0.0	0.0	0.750	0.740	0.726

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.798	0.820	0.817	0.798	0.783	0.763	0.733
2	0.750	0.785	0.811	0.817	0.802	0.786	0.760
3	0.714	0.743	0.787	0.806	0.807	0.792	0.770
4	0.0	0.720	0.760	0.789	0.798	0.786	0.767
5	0.0	0.0	0.719	0.757	0.769	0.769	0.755
6	0.0	0.0	0.711	0.729	0.748	0.749	0.739
7	0.0	0.0	0.0	0.701	0.729	0.733	0.727
8	0.0	0.0	0.0	0.664	0.683	0.697	0.695
9	0.0	0.0	0.0	0.0	0.651	0.675	0.684
10	0.0	0.0	0.0	0.0	0.629	0.655	0.663

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =

0.900	0.780	0.660	0.530	0.420	0.280	0.160	0.130
0.060	0.030	0.015	0.008	0.004	0.002	0.001	

A=4.000 B=3.000 G=0.600 H= 8.00 X1=12.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.50	2.86	4.91	7.84	11.66	16.29	21.16
2	1.66	2.86	4.67	7.23	10.13	14.09	17.73
3	1.90	2.84	4.58	7.08	10.17	14.23	17.73
4	0.0	2.94	4.39	6.81	10.00	14.26	17.70
5	0.0	3.40	4.34	6.93	10.49	14.91	18.33
6	0.0	0.0	4.33	6.66	10.25	14.84	18.30
7	0.0	0.0	4.61	6.15	9.58	14.28	17.84
8	0.0	0.0	0.0	6.23	10.09	15.01	18.55
9	0.0	0.0	0.0	6.52	9.56	14.60	18.25
10	0.0	0.0	0.0	6.91	9.17	14.31	18.06

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.828	0.790	0.770	0.757	0.746	0.732	0.706
2	0.822	0.790	0.771	0.759	0.750	0.739	0.725
3	0.811	0.791	0.772	0.760	0.750	0.739	0.725
4	0.0	0.789	0.773	0.761	0.750	0.739	0.725
5	0.0	0.783	0.774	0.760	0.749	0.737	0.722
6	0.0	0.0	0.774	0.761	0.750	0.737	0.723
7	0.0	0.0	0.772	0.764	0.751	0.739	0.725
8	0.0	0.0	0.0	0.763	0.750	0.736	0.721
9	0.0	0.0	0.0	0.762	0.752	0.738	0.723
10	0.0	0.0	0.0	0.760	0.753	0.739	0.724

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.789	0.800	0.802	0.798	0.790	0.776	0.749
2	0.747	0.770	0.787	0.804	0.798	0.787	0.771
3	0.711	0.733	0.776	0.789	0.797	0.785	0.767
4	0.0	0.714	0.752	0.773	0.778	0.779	0.762
5	0.0	0.685	0.715	0.750	0.759	0.752	0.744
6	0.0	0.0	0.685	0.720	0.736	0.737	0.726
7	0.0	0.0	0.653	0.689	0.719	0.722	0.712
8	0.0	0.0	0.0	0.645	0.679	0.690	0.690
9	0.0	0.0	0.0	0.624	0.644	0.668	0.673
10	0.0	0.0	0.0	0.596	0.617	0.650	0.656

*** PARAMETERS OF FERTILITY ***

STOPPING RULE =
0.900 0.780 0.660 0.530 0.420 0.280 0.180 0.130
0.060 0.030 0.015 0.008 0.004 0.002 0.001

A=5.500 B=2.500 G=0.400 H= 4.00 X1=14.00

TIME EXPOSURE TO THE RISK OF DYING

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	1.03	1.99	3.25	4.90	8.00	12.94	17.95
2	1.40	2.25	3.53	4.99	7.14	9.92	14.57
3	1.75	2.37	3.67	5.35	7.53	10.15	13.97
4	0.0	2.83	3.63	5.39	7.67	10.53	14.27
5	0.0	0.0	3.95	5.64	8.31	11.91	15.68
6	0.0	0.0	4.22	5.45	8.23	12.11	15.97
7	0.0	0.0	0.0	5.81	7.75	11.54	15.52
8	0.0	0.0	0.0	0.0	8.19	12.69	16.71
9	0.0	0.0	0.0	0.0	8.53	12.24	16.43
10	0.0	0.0	0.0	0.0	9.00	11.92	16.28

STANDARD PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.848	0.807	0.785	0.770	0.756	0.743	0.724
2	0.833	0.802	0.782	0.769	0.760	0.750	0.738
3	0.818	0.800	0.780	0.767	0.758	0.750	0.740
4	0.0	0.790	0.780	0.767	0.758	0.749	0.739
5	0.0	0.0	0.777	0.766	0.755	0.745	0.734
6	0.0	0.0	0.775	0.767	0.756	0.745	0.733
7	0.0	0.0	0.0	0.765	0.757	0.746	0.734
8	0.0	0.0	0.0	0.0	0.756	0.743	0.730
9	0.0	0.0	0.0	0.0	0.755	0.745	0.731
10	0.0	0.0	0.0	0.0	0.753	0.745	0.732

OBSERVED PROPORTIONS OF SURVIVING CHILDREN

ORDER	15-19	20-24	25-29	30-34	35-39	40-44	45-49
1	0.816	0.822	0.825	0.818	0.804	0.789	0.770
2	0.776	0.791	0.812	0.821	0.812	0.798	0.774
3	0.741	0.759	0.791	0.808	0.810	0.797	0.781
4	0.0	0.749	0.767	0.792	0.798	0.792	0.775
5	0.0	0.0	0.750	0.766	0.774	0.773	0.762
6	0.0	0.0	0.724	0.740	0.756	0.755	0.745
7	0.0	0.0	0.0	0.727	0.734	0.738	0.730
8	0.0	0.0	0.0	0.0	0.699	0.710	0.708
9	0.0	0.0	0.0	0.0	0.681	0.691	0.691
10	0.0	0.0	0.0	0.0	0.663	0.671	0.676

APPENDIX 3

Number of Cases and Standard Deviations for Observed Time-Exposures

Table A.7.1 MEXICO: Number of cases and standard deviation for the average exposure to risk by mother's age and parity. (W.F.S.)

Parity order	Age Group 15-19	20-24	25-29	34-34	35-39	40-44	45-49
	Number of cases and standard deviations						
1	194 0.95	329 1.40	142 2.77	68 4.46	48 6.42	28 7.12	35 7.94
2	109 0.75	303 1.02	215 2.03	114 3.72	55 5.26	54 5.97	27 6.50
3	24 0.71	227 1.16	241 1.92	127 2.81	87 4.18	47 5.59	43 6.24
4	4 0.71	97 0.99	202 1.67	155 2.57	96 4.42	56 4.73	53 5.05
5		38 1.00	176 1.58	148 2.17	89 3.37	67 4.58	50 4.16
6		8 0.68	103 1.40	134 2.07	126 2.70	78 3.12	52 3.99
7		4 0.58	46 1.35	117 1.91	132 2.57	74 3.40	65 4.29
8			16 1.04	86 1.66	109 2.31	82 2.89	52 3.79
9			6 0.37	44 1.26	87 2.29	74 2.52	78 3.22
10				27 1.27	73 2.08	54 2.65	53 3.03

Table A.7.2 **PERU: Number of cases and standard deviation for the average exposure to risk by mother's age and parity. (W.F.S.)**

Parity order	15-19	20-24	25-29	34-34	35-39	40-44	45-49
				Number of cases and standard deviations			
1	142 0.87	274 1.44	126 2.59	60 4.85	42 6.83	31 6.81	24 6.76
2	68 0.70	281 1.27	203 1.88	121 3.72	62 5.04	34 6.02	45 6.29
3	14 0.59	174 1.01	223 2.00	151 2.72	103 4.18	79 5.01	47 4.93
4	1 0.00	67 1.13	206 1.59	143 2.41	94 3.57	81 4.16	51 5.37
5		24 0.96	134 1.32	134 2.29	110 3.40	72 3.68	70 4.11
6		5 0.93	72 1.07	105 2.10	119 3.03	83 4.21	77 4.10
7			30 1.17	99 1.53	102 2.58	91 3.17	74 3.93
8			7 0.60	59 1.58	97 2.35	84 3.31	67 4.04
9			3 1.46	26 1.46	80 1.65	83 2.51	66 2.55
10				6 1.32	55 2.13	58 2.48	50 3.91

Table A.7.3 COLOMBIA: Number of cases and standard deviation for the average exposure to risk by mother's age and parity. (W.F.S.)

Parity order	Age Group 15-19	20-24	25-29	34-34	35-39	40-44	45-49
	Number of cases and standard deviations						
1	99 0.87	219 1.65	117 3.33	42 4.88	40 6.76	22 6.68	18 6.87
2	49 0.72	193 1.38	154 2.31	69 3.74	47 5.70	30 5.51	18 5.79
3	9 0.90	101 1.21	132 2.22	102 2.97	61 4.29	43 4.72	31 5.90
4	2 0.43	40 1.24	92 2.00	80 2.70	60 3.90	34 4.52	36 6.04
5		13 0.98	66 1.55	91 2.47	53 3.90	41 3.90	35 5.21
6		4 0.72	43 1.39	52 2.13	46 2.58	35 4.41	32 3.81
7			22 1.46	37 2.18	64 2.75	47 3.84	30 4.61
8				30 1.70	47 2.49	43 3.29	32 3.29
9				21 1.77	33 2.22	33 2.86	29 3.58
10					18 2.21	33 3.16	33 2.95

Table A.7.4 LESOTHO: Number of cases and standard deviation for the average exposure to risk by mother's age and parity. (W.F.S.)

Parity order	Age Group						
	15-19	20-24	25-29	34-34	35-39	40-44	45-49
	Number of cases and standard deviations						
1	143 0.85	300 1.51	90 3.14	37 5.03	35 5.02	35 6.76	20 5.05
2	21 1.01	234 1.08	184 2.26	51 4.15	36 4.12	46 6.24	28 6.10
3		63 1.12	197 1.64	95 3.28	57 4.10	50 5.00	28 6.91
4		22 1.66	121 1.51	111 1.62	68 3.46	51 3.93	30 5.00
5		11 1.61	33 1.50	109 1.72	77 3.10	50 4.47	31 4.79
6			9 2.40	54 1.98	75 2.55	59 3.07	33 3.13
7			2 1.14	24 1.34	60 1.49	61 3.33	31 3.60
8				10 1.15	35 1.82	51 1.96	26 3.00
9				4 1.45	12 2.53	37 1.97	21 2.67
10					6 1.27	18 2.26	14 2.63

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